Apportionment and Proportionality: A Measured View

P Kestelman
No email available.

1 Introduction

Collins (2003) **English Dictionary** defined 'Proportional Representation' (PR) as: "representation of parties in an elective body in proportion to the votes they win". Few elections translate every Party Vote-fraction into the same Seat-fraction, thereby mediating exact PR; and raising the question of when to describe an election as full PR, semi-PR ('broad PR') or non-PR.

According to Gallagher, Marsh and Mitchell [11], "Ireland uses the system of proportional representation by means of the single transferable vote (PR-STV) at parliamentary, local, and European Parliament elections (the president, too is elected by the single transferable vote)". Presidential single-member STV is **Alternative Voting** (AV), which also elects the Australian House of Representatives.

Is AV therefore a PR electoral system? The Independent Commission on the Voting System [13] — the Jenkins Report — maintained that AV alone "is capable of substantially adding to ['First-Past-the-Post' (FPP)] disproportionality". The more recent Independent Commission on PR [12] affirmed that "AV can produce a hugely disproportionate result".

How should we compare the Party disproportionality of different electoral systems? Which is the fairest — most proportional — electoral system? In other words, how should disproportionality — departure from exact PR — be quantified?

2 Apportionment

First consider the analogous question of the fairest method of apportionment. Collins (2003) **English Dictionary** defined 'apportionment' as: "*U.S. government.* the proportional distribution of the seats in a legislative body, esp. the House of Representatives, on the basis of population".

The USA has long wrestled with the problem of the most representative apportionment; trying various methods (Balinski and Young [1]). **Table 4.1** gives the apportionment of 105 Seats among 15 States in the first (1791) House of Representatives, applying the main five Divisor methods. For the five most and least populous States, proportionality is measured as the ratio between their aggregate Seat-fractions and Population-fractions ($S\%/P\%$).

Adams, Dean and Hill yield the same apportionment: slightly *under*-representing the five most populous States ($S\%/P\% = 0.99$); while *over*-representing the five least populous States ($S\%/P\% = 1.09$). These methods produce a **Relative Bias** of + 10 percent (Bottom/Top third $S\%/P\% = 1.09/0.99 = 1.10$).

On the other hand, Jefferson *over*-represents the top five States ($S\%/P\% = 1.02$); and *under*-represents the least populous States ($S\%/P\% = 0.89$): a Relative Bias of – 13 percent (Bottom/Top third $S\%/P\% = 0.89/1.02 = 0.87 = 1 - 0.13$). With the lowest Relative Bias (– 2 percent), Webster yields the fairest 1791 Apportionment.

Requiring at least one Seat per State usually over-represents the least populous States. Eliminating that constraint — so quantifying method-specific bias more precisely — **Table 4.1** (bottom panel) gives the Mean Bias for all 22 USA apportionments (1791–2000). The Webster (Sainte-Laguë) Method proved the least biased overall (averaging 0.1 percent); whereas Adams (Smallest Divisor) and Jefferson (d'Hondt) were the most biased (over 20 percent).

3 Apportioning England

Nearer home, **Table 4.2** apportions 71 MEPs between the nine English Regions, applying the five Divisor methods to their 1999 Electorates. Adams and Dean co-

incided but, despite identical Bottom/Top third Relative Bias, differed slightly from Hill and Webster. Which apportionment is fairer?

The *European Parliament (Representation) Act 2003* prescribes that: "the ratio of electors to MEPs is as nearly as possible the same in each electoral region". In testing fairness, the Electoral Commission [7] accepted a measure that "involves calculating for each region the difference between the number of electors per MEP for that region and the overall number of electors per MEP, and adding up all these differences (having ignored minus signs). The smaller this total is, the more equitable the outcome".

A little mathematical notation helps here. The overall number of Electors per MEP, $E/S = \sum E_R/S_R$, where $\sum$(Sigma) denotes 'Sum' (over all Regions); E_R is the number of electors in a Region; and S_R is the corresponding number of seats. Each Regional deviation is the absolute difference (that is, ignoring negative signs) between its E_R/S_R and E/S ; and

$$\begin{aligned}\text{Total Deviation} &= \sum \mid E/S - E_R/S_R \mid \\ &= E/S \sum \mid 1 - (E_R/E)/(S_R/S) \mid \\ &= E/S \sum \mid 1 - E_R\%/S_R\% \mid ,\end{aligned}$$

where E_R % and S_R % are the Regional Elector- and Seat- fractions *(percent)*, respectively.

For any given apportionment, total Electors and Seats — and thus E/S — are fixed: hence Regional MEP apportionment is required to minimise $\sum \mid 1 - E_R\%/S_R\% \mid$. The UK statutory criterion implies the Dean Method (Balinski and Young [1]).

Nonetheless, for the June 2004 European Elections, the Electoral Commission [7] recommended the Webster (Sainte-Laguë) Method, making the ratio of *MEPs to electors* as nearly as possible the same in each Region (beyond the statutory minimum of three MEPs). Based on December 2002 Regional electorates, Dean and Webster apportionments coincided.

We may therefore define a **Dean Index** $= \sum \mid 1 - E_R\%/S_R\% \mid$; and a **Webster Index** $= \sum \mid 1 - S_R\%/E_R\% \mid$. **Table 4.2** (bottom panel) confirms that the Dean Method minimises the Dean Index; and the Webster Method minimises the Webster Index.

4 Paradox and Proportionality

Overall measures of malapportionment (like the Dean and Webster indices defined above) are better than partial measures (like Bottom/Top third Relative Bias). The Webster Method minimises total *relative* differences between Regional Elector-fractions and Seat-fractions:

$$\begin{aligned}\text{Webster Index} &= \sum \mid 1 - S_R\%/E_R\% \mid \\ &= \sum \mid E_R\% - S_R\% \mid /E_R\%.\end{aligned}$$

Total *absolute* differences between Regional Elector-fractions and Seat-fractions are minimised by the **Hamilton** Method (Largest Remainders: LR–Hare).

This Quota Method allocates to each Region the integer part of its proportional entitlement (number of Hare Quotas: one Hare Quota = National Electors/National Seats). Any residual seats are then allocated to the regions with the largest fractional parts (remainders) of Hare Quotas.

We may therefore define a **Hamilton Index** $= \sum \mid E_R\% - S_R\% \mid$; minimised by the Hamilton Method. Applied to all 22 USA apportionments (without seat minima), Hamilton averages a (Bottom/Top third) Relative Bias of – 0.3 percent: differing insignificantly from Webster (– 0.1 percent).

Unlike Webster, the Hamilton Method of apportionment is vulnerable to paradox: notably the Alabama Paradox. The 1880 USA Census disclosed that, if total House size were *increased* from 299 to 300 seats, then the Hamilton apportionment to Alabama would have *decreased* from eight to seven seats (Balinski and Young [1])!

That consideration excludes Hamilton as a method of apportionment; though not necessarily for evaluating malapportionment. So how best to quantify malapportionment — or disproportionality?

5 Party Disproportionality

Gallagher [10] concluded that each PR method "minimizes disproportionality according to the way it defines disproportionality". However, Lijphart [14] argued that LR-Hare (Hamilton) and Sainte-Laguë (Webster) mediate "inherently greater proportionality" than d'Hondt (Jefferson); thereby justifying proportionality measures "biased in favour of LR-Hare".

LR-Hare minimises the **Loosemore-Hanby Index** (Loosemore and Hanby, [15]):

LHI *(percent)*$= \frac{1}{2} \sum \mid V_P\% - S_P\% \mid$,
where $V_P\%, S_P\% =$ Party Vote–, Seat–fractions *(percent)*.

Compare the Hamilton Index $= \sum \mid E_R\% - S_R\% \mid$, as defined above. Halving the sum ensures that LHI ranges 0–100 percent.

LHI is the 'DV score' mentioned by the Independent Commission on the Voting System [13]; and as defined by the Independent Commission on PR [12]. LHI complements the **Rose Proportionality Index** (Mackie and Rose, [16]) *percent*:

$= 100 - \frac{1}{2} \sum \mid V_P\% - S_P\% \mid = 100 -$LHI *(percent)*.

Table 4.3 illustrates the calculation of LHI and RPI for the 2004 European Parliamentary Election in Britain. Over-represented and under-represented Party Total Deviations are necessarily equal and opposite (± 14.7 percent in **Table 4.3**); and Party total over-representation is simply the Loosemore-Hanby Index (LHI = 14.7 percent).

6 Debate

As a measure of Party disproportionality, the Loosemore-Hanby Index (LHI) has been criticised on three main grounds: for violating Dalton's Transfer Principle (Taagepera and Shugart [22]); for being vulnerable to paradox (Gallagher [10]); and for exaggerating the disproportionality of PR systems involving many parties (Lijphart [14]).

Dalton's Transfer Principle states that transferring wealth from a richer to a poorer person decreases inequality, decreasing any inequality index (Taagepera and Shugart [22]). However, transferring seats between over-represented parties (or between under-represented parties) leaves LHI unchanged.

Thus in the 2004 European Election in Britain (**Table 4.3**), imagine the Conservatives (from 27 to 25 seats) losing two seats to Labour (from 19 to 21 seats). Then both Party deviations would converge ($S_P\% - V_P\% =$ from + 9.3 to + 6.6 percent, and from + 2.7 to + 5.4 percent, respectively); decreasing GhI (from 8.3 to 7.7 percent), leaving LHI unchanged (14.7 percent). However, Party total over-representation remains unchanged: so why should overall disproportionality change?

Minimised by LR-Hare (Hamilton), LHI is susceptible to the paradoxes of that Quota method (Gallagher [10]). Because Sainte-Laguë (Webster) is the least biased Divisor method — and immune to paradox — Gallagher [10] recommended a **Sainte-Laguë Index** "as the standard measure of disproportionality":

SLI *(percent)* $= \sum (V_P\% - S_P\%)^2 / V_P\%$.

However, in a single-member constituency, if the winner receives under half of all votes, then SLI exceeds 100 percent (unlike LHI, which measures unrepresented — wasted — votes).

Nowadays, Gallagher [10] is mainly cited for his 'Least Squares Index':

GhI *(percent)*$= \sqrt{\frac{1}{2} \sum (V_P\% - S_P\%)^2}$.

Also minimised by LR–Hare, GhI is subject to the same paradoxes as LHI. Gallagher [10] saw GhI as "a happy medium" between LHI and the **Rae Index** (Rae [18]):

RaI *(percent)* $= \sum \mid V_P\% - S_P\% \mid / N$,
where N = Number of parties ($V_P\% > 0.5$ percent).

Thus RaI measures *average* deviation per Party; whereas LHI measures (half) *Total* Deviation. Yet why hybridise such conceptually distinct measures in one measure (GhI)?

Taagepera and Grofman [21] have attributed the recent shift, from LHI towards GhI, "to sensitivity to party system concentration"; based on the intuition of Lijphart [14] that a few large deviations ($V_P\% - S_P\%$) should be evaluated as more disproportional overall than many small deviations with the same Total Deviation (and hence LHI). It remains unclear why larger Party deviations should be potentiated; and smaller ones attenuated.

For example, in the 2004 European Election in Britain, exact GhI was 8.3 percent. However, aggregating unrepresented parties ($S_P\% = 0.0$ percent: **Table 4.3**) increases GhI to 10.7 percent; leaving LHI unchanged (14.7 percent). In the process, Party total under-representation has not changed: so why should Total Disproportionality change? Likewise, in single-member constituencies, GhI depends on the division of votes among losing candidates.

Monroe [17] proposed an inequity index rather similar to GhI:

$$\mathbf{MrI}(percent) = \sqrt{\frac{\sum (V_P\% - S_P\%)^2}{1 + \sum (V_P\%/100)^2}}$$

LR-Hare also minimises MrI; which falls below 100 percent for extreme disproportionality involving more than two parties (like GhI, but unlike LHI).

Taagepera and Shugart [22] mentioned an electoral analogue of the widespread Gini Inequality Index, with several examples; but without defining any **Gini Disproportionality Index** (GnI). It turns out that **GnI** *(percent)*:

$$= \sum\sum \mid (V_P\% \times S_Q\%) - (S_P\% \times V_Q\%) \mid /200$$

Thus GnI sums the absolute differences between the $S_P\%/V_P\%$ ratios of every pair of parties, weighted by the product of their vote-fractions $(V_P\%/100)$. This complex GnI satisfies Dalton's Transfer Principle; and aggregating unrepresented parties ($S_P\%$ = 0.0 percent) leaves GnI unchanged (like LHI and SLI).

Taagepera and Grofman [21] evaluated 19 disproportionality indices against 12 criteria, sustaining five measures: LHI; GhI; SLI ('chi-square'); MrI; and GnI. Nonetheless, they overlooked both a Farina Index (FrI) and a Borooah Index (BrI).

Woodall [24] cited JEG Farina for a vector-based measure of Party Total Disproportionality: the angle between two multidimensional vectors, whose coordinates are Party vote and seat numbers. Its fraction of a right angle defines a **Farina Index**, FrI (*percent*) =

$$\arccos\left[\frac{\sum(S_P\% \times V_P\%)}{\sqrt{\sum S_P\%^2 \times \sum V_P\%^2}}\right] \times 100/90^\circ$$

ranging 0–100 percent (instead of 0–90 degrees).

Borooah [2] proposed an electoral analogue of the Atkinson Inequality Index, depending on "society's aversion to inequality" (like Gini, originally measuring income inequality). Establishing national 'Societal Aversion to Disproportionality' seems arbitrary; while a moderate value (SAD = 2) defines a **Borooah Index**,

BrI *(percent)* $= 100 - 1/[\sum(S_P\%/100)^2/V_P\%]$, ranging 0–100 percent.

7 Correlations

For 82 general elections in 23 countries (1979–89), Gallagher [10] reported high correlations between LHI, GhI and SLI. Graphing high correlations between LHI, GhI, SLI and FrI, Wichmann [23] noted that central placement reinforced LHI.

Table 4.4 gives the correlations between all seven indices in the last 44 UK general elections (1832–2005). Most notably, LHI proved very highly correlated with GnI; GhI with MrI; and SLI with BrI ($R > 0.99$). Indeed, LHI and GnI were highly correlated ($R > 0.95$) with all other measures of Party Total Disproportionality.

8 Proportionality Criteria

The Independent Commission on the Voting System [13] observed that "full proportionality ... is generally considered to be achieved as fully as is normally practicable if [LHI%] falls in the range of 4 to 8". More generously, we might allow LHI under 10 percent to characterise **full PR**. LHI ranging 10–15 percent could then encompass **semi-PR** ('broad PR'); with LHI over 15 percent constituting **non-PR**.

In **UK** general elections (FPP) since World War I, LHIs have ranged from 27 percent (1918); to only four percent (1951) — ironically, when the Conservatives won fewer votes, but more seats, than Labour (Rallings and Thrasher [19]). In the last nine general elections (1974–2005: **Table 4.5**), LHIs have ranged 15–24 percent, averaging 20 percent: clearly *non-PR*.

What of the nominally PR elections, introduced in Britain since 1997? In the 1999 and 2004 European Parliamentary elections, Regional d'Hondt yielded LHIs of 14.1–14.7 percent (between Party List votes and MEPs) nationwide: barely *semi-PR*. Likewise applied regionally, either Sainte-Laguë (LHI = 6.1–8.4 percent), or LR–Hare (LHI = 6.1–5.4 percent), would have mediated *full PR*. So the method used here can make a considerable difference.

In the 1999 and 2003 **Scottish Parliament** and **National Assembly for Wales** elections, between Party List votes and Total (FPP Constituency + Additional Regional) seats, LHIs ranged 11–14 percent. The 2000 and 2004 **London Assembly** elections (also FPP-plus, but with a five percent Party Vote Threshold) yielded similar Party List LHIs of 14–15 percent. Thus all three British Regional Assemblies remain *semi-PR* at best (Independent Commission on PR [12]).

In contrast, both 1998 and 2003 **Northern Ireland Assembly** elections (multi-member STV) mediated First Preference LHIs of only 6.0–6.4 percent: *full PR*. **Table 4.6** ranks UK national and regional election LHIs over the past decade (1995–2005).

9 Vote Transferability and District Magnitude

Transferable voting complicates evaluating the disproportionality of both AV and multi-member STV. First Count LHI is *not* the sole criterion; though Final Count LHI over-estimates Party proportionality (Gallagher [9]). For comparing transferable voting with

other electoral systems, averaging First and Final Count LHIs appears reasonable.

Under Alternative Voting (**AV**), in the last nine general elections in **Australia** (1983–2004), First Count LHI ranged 12–20 percent, averaging 16 percent (**Table 4.5**): practically *non-PR*. Final Count LHI ranged 5–13 percent, averaging eight percent (PR); while mean First + Final Count LHI averaged 12 percent: *semi-PR* overall (compare **Table 4.6**).

So much for empirical claims that AV "is capable of substantially adding to [FPP] disproportionality" (Independent Commission on the Voting System, [13]). FPP votes — involving tactical considerations — should not only be compared with AV First Preferences.

Taagepera and Shugart [22] called AV 'semi-PR'; and attributed any 'semi-PR effect' in multi-member STV elections to low **District Magnitude** (M = Number of Seats per Constituency). As Gallagher [9] noted: "the smaller the constituency [M], the greater the potential for disproportionality"; and reported decreasing LHI with increasing STV District Magnitude in 16 Irish general elections (1927–1973).

Table 4.7 gives national aggregate LHI, by District Magnitude and Count, in the last 13 Irish general elections (1961–2002). Between such low District Magnitudes (M = 3–5), disproportionality might be expected to fall steeply: so the relative insignificance of all LHI differences is remarkable.

Overall, First Count LHIs ranged 3–13 percent (averaging seven percent); Final Count LHIs ranged 1–7 percent (averaging three percent); and mean First + Final Count LHI averaged only five percent (and 6–7 percent for M = 3–5). Virtually regardless of District Magnitude, multi-member STV mediates *full PR*.

10 Conclusions

Sainte-Laguë (Webster) is the most equitable method of apportionment — and the most proportional electoral principle. The d'Hondt (Jefferson) Method overrepresents the most populous regions (and the most popular parties).

Not much has changed since Gallagher [10] lamented "surprisingly little discussion of what exactly we mean by proportionality and how we should measure it". Certainly, Party disproportionality indices have proliferated; among which the Loosemore-Hanby Index (LHI) — straightforwardly measuring Party total overrepresentation — remains the most serviceable. Moreover, such absolute disproportionality is what matters politically [14, 21].

Continuing debate on the 'best' measure of disproportionality may distract attention from the main task of evaluating the relative disproportionality of different electoral systems. Taagepera and Grofman [21] marginally preferred the Gallagher Index (GhI); allowing that its advantages over LHI were debatable.

LHI fails Dalton's Transfer Principle; yet transferring seats between over- (or under-) represented parties should arguably not change a measure of Total Disproportionality. LHI, GhI and MrI alike remain vulnerable to the paradoxes of the Largest Remainders (LR-Hare/Hamilton) Method.

The Sainte-Laguë Index (SLI) is unsuitable for measuring Party Total Disproportionality. Fortunately highly correlated with LHI, the Gini Disproportionality Index (GnI) is rather complicated to explain and calculate (virtually necessitating computerisation). Interestingly, Riedwyl and Steiner [20] traced the LHI concept back to Gini (1914–15).

Settling for the most elementary LHI clearly demonstrates that, in recent UK general elections, FPP has proved *non-PR*. Even nominally PR elections in Britain have barely mediated *semi-PR*. Yet in both Northern Ireland Assembly elections, multi-member Single Transferable Voting has yielded *full PR* of Party First Preferences.

Allowing for vote transferability, STV has also mediated full PR in recent Irish general elections; hardly affected by District Magnitude (between three and five seats per constituency). Likewise in Australia, Alternative Voting has arguably proved semi-PR; and certainly no more disproportional than First-Past-the-Post.

11 References

[1] Balinski, M. and Young, H. P. (2001): *Fair Representation*. Meeting the Ideal of One Man, One Vote. Brookings Institution Press, Washington DC.

[2] Borooah, V. K. (2002): 'The Proportionality of Electoral Systems: Electoral Welfare and Electoral Inequality': *Economics and Politics* **14**, 83–98.

[3] Chief Electoral Officer for Northern Ireland (1998): *The New Northern Ireland Assembly Election 25 June 1998: Election of 6 Members to*

each of the 18 (NI) Westminster Constituencies. Belfast.

[4] Dáil Éireann (1962–2003): *Election Results and Transfer of Votes in General Election*: October 1961 – May 2002. Stationery Office, Dublin.

[5] Electoral Commission (2001): *Election 2001: The Official Results*. London.

[6] Electoral Commission (2003a): *Combination of Gibraltar with a European electoral region in England and Wales*. Consultation paper, June 2003.

[7] Electoral Commission (2003b): *Distribution between electoral regions of UK MEPs*. Recommendation, October 2003.

[8] Electoral Office for Northern Ireland (2004): *Northern Ireland Assembly Elections 26 November 2003 Results*.

[9] Gallagher, M. (1986): 'The Political Consequences of the Electoral System in the Republic of Ireland': *Electoral Studies 5*, 253–75.

[10] Gallagher, M. (1991): 'Proportionality, Disproportionality and Electoral Systems': *Electoral Studies 10*, 33–51.

[11] Gallagher, M, Marsh, M. and Mitchell, P. *eds* (2003): *How Ireland Voted 2002*. Palgrave Macmillan, Basingstoke.

[12] Independent Commission on PR (2004): *Changed Voting Changed Politics*. Lessons of Britain's Experience of PR since 1997. Constitution Unit, School of Public Policy, UCL, London.

[13] Independent Commission on the Voting System (1998): *The Report of the Independent Commission on the Voting System*. Cm 4090–I. The Stationery Office, London.

[14] Lijphart, A. (1994): *Electoral Systems and Party Systems*. Oxford University Press.

[15] Loosemore, J. and Hanby, V. (1971): 'The Theoretical Limits of Maximum Distortion: Some Analytic Expressions for Electoral Systems': *British Journal of Political Science 1*, 467–77.

[16] Mackie, T. T. and Rose, R. (1991): *The International Almanac of Electoral History*. Macmillan, Basingstoke.

[17] Monroe, B. L. (1994): 'Disproportionality and Malapportionment: Measuring Electoral Inequity': *Electoral Studies* **13**, 132–49.

[18] Rae, D. (1967): *The Political Consequences of Electoral Laws*. Yale University Press, New Haven.

[19] Rallings, C. and Thrasher, M. *eds* (2000): *British Electoral Facts 1832–1999*. Ashgate, Aldershot.

[20] Riedwyl, H. and Steiner, J. (1995): 'What is Proportionality Anyhow?': *Comparative Politics* **27**, 357–369.

[21] Taagepera, R and Grofman, B (2003): 'Mapping the Indices of Seats-Votes Disproportionality and Inter-Election Volatility': *Party Politics* **9**, 659–677.

[22] Taagepera, R. and Shugart, M. S. (1989): *Seats and Votes*. The Effects and Determinants of Electoral Systems. Yale University Press, New Haven.

[23] Wichmann, B. A. (2003): 'Proportionality Revisited': *Voting matters* Issue 16, 6–8.

[24] Woodall, D. R. (1986): 'How Proportional is Proportional Representation? *The Mathematical Intelligencer* **8** : 4, 36–46.

Table 4.1: State Population, Seat Apportionment and Relative Bias

(**Bottom/Top third** most populous States), by Divisor Method: House of Representatives, **USA:** 1791 Apportionment; and 1791–2000 **Mean Bias** (22 Apportionments, without seat minima).

State of Union	Population	**Divisor Method: Number of Seats (S)**				
	(P)	Adams	Dean	Hill	Webster	Jefferson
Total (USA)	3,615,920	105	105	105	105	105
Virginia	630,560	18	18	18	18	19
Massachusetts	475,327	14	14	14	14	14
Pennsylvania	432,879	12	12	12	13	13
North Carolina	353,523	10	10	10	10	10
New York	331,589	10	10	10	10	10
Maryland	278,514	8	8	8	8	8
Connecticut	236,841	7	7	7	7	7
South Carolina	206,236	6	6	6	6	6
New Jersey	179,570	5	5	5	5	5
New Hampshire	141,822	4	4	4	4	4
Vermont	85,533	3	3	3	2	2
Georgia	70,835	2	2	2	2	2
Kentucky	68,705	2	2	2	2	2
Rhode Island	68,446	2	2	2	2	2
Delaware	55,540	2	2	2	2	1
Top third (5)	2,223,878	64	64	64	65	66
Bottom third (5)	349,059	11	11	11	10	9
Seat/ Population	Top third	0.99	0.99	0.99	1.01	1.02
fraction (S%/P%)	Bottom third	1.09	1.09	1.09	0.99	0.89
1791 **Relative Bias**, *percent* *		+10	+10	+10	–2	–13
1791–2000 **Mean Bias**, *percent* *		+20.3	+7.0	+ 5.0	–0.1	–20.7

* **Relative Bias:** Percentage deviation from unity of ratio between Seat/Population (or $S\%/P\%$) ratios of **Bottom/Top third** most populous States.

Data Source: Balinski and Young [1].

Table 4.2: Regional Electors, Seat Apportionment and Relative Bias

(**Bottom/Top third** most populous Regions) and **Malapportionment Index**, by Divisor Method: MEPs, **England**, 1999.

Region	Electors (E)	**Divisor Method: Number of Seats (S)**				
		Adams	Dean	Hill	Webster	Jefferson
Total (England)	37,079,720	71	71	71	71	71
South East	6,023,991	11	11	12	12	12
North West	5,240,321	10	10	10	10	10
London	4,972,495	10	10	9	9	10
Eastern	4,067,524	8	8	8	8	8
West Midlands	4,034,992	8	8	8	8	8
Yorkshire & Humber	3,795,388	7	7	7	7	7
South West	3,775,332	7	7	7	7	7
East Midlands	3,199,711	6	6	6	6	6
North East	1,969,966	4	4	4	4	3
Top third (3)	16,236,807	31	31	31	31	32
Bottom third (3)	8,945,009	17	17	17	17	16
Seat-/Electorate- Top third		0.997	0.997	0.997	0.997	1.029
fraction ($S\%/E\%$) Bottom third		0.993	0.993	0.993	0.993	0.934
Relative Bias, *percent* *		–0.46		–0.46		–9.24
Malapportionment Dean		30.96		30.98		50.01
Index *(percent)* † Webster		31.22		31.07		45.05

* **Relative Bias:** Percentage deviation from unity of ratio between Seat/Electorate (or $S\%/E\%$) ratios of Regions with **Bottom/Top third** most electors.

† **Malapportionment Index:**
Dean Index *(percent)* $= \sum \mid 1 - E_R\%/S_R\% \mid \times 100$; and
Webster Index *(percent)* $= \sum \mid 1 - S_R\%/E_R\% \mid \times 100$:

Data Source: Electoral Commission [6].

Table 4.3: Analysis of Party Votes and Seats

Number, Fraction and **Loosemore-Hanby Index**: European Election (d'Hondt Regional Closed Party Lists): **Britain**, June 2004.

Party	Number		Fraction, *percent*		Seat–Vote Fraction
	Votes (V_P)	Seats (S_P)	Votes $(V_P\%)$	Seats $(S_P\%)$	**Deviation**, *percent* $(S_P\% - V_P\%)$ *
Total (Britain)	16,448,605	75	100.0	100.0	0.0
Conservative	4,397,090	27	26.7	36.0	+9.3
Labour	3,718,683	19	22.6	25.3	+2.7
UK Independence	2,650,768	12	16.1	16.0	–0.1
Liberal Democrat	2,452,327	12	14.9	16.0	+1.1
Green	1,028,283	2	6.3	2.7	–3.6
Scottish National	231,505	2	1.4	2.7	+1.3
Plaid Cymru	159,888	1	1.0	1.3	+0.4
Others (unrepresented)	1,810,061	0	11.0	0.0	–11.0
Over-represented *	10,959,493	61	66.6	81.3	+14.7†
Under-represented	5,489,112	14	33.4	18.7	–14.7

* Over-represented Party $S_P\% > V_P\%$ (under-represented $S_P\% < V_P\%$).

† **Loosemore-Hanby Index** (LHI) = Party total over-representation
$= \frac{1}{2} \sum \mid V_P\% - S_P\% \mid =$ **14.7 percent**.
Rose Proportionality Index (RPI) = Complement of Party total over-representation = 100.0 – 14.7 = **85.3 percent**.

Data Source: Guardian, 16 June 2004.

Table 4.4: Correlations between Seven Party Total Disproportionality Indices

UK (FPP: 44 general elections), 1832–2005.
Values as percentages.

Index	LHI	GhI	SLI	MrI	GnI	FrI	BrI
LHI	–	96.4	91.0	97.7	98.1	96.5	91.4
GhI		–	84.8	99.8	94.0	96.4	86.1
SLI			–	86.4	92.3	84.7	99.5
MrI				–	95.4	97.2	87.5
GnI					–	94.7	93.0
FrI						–	85.6
Mean Index	11.5	9.2	11.4	11.2	13.4	11.8	9.6

Data sources: Electoral Commission [5]; Rallings and Thrasher [19] and Guardian, 7 May 2005.

Table 4.5: Loosemore-Hanby Index

Last Nine General Elections in **UK** (FPP), 1974–2005;
and **Australia** (AV), 1983–2004.

UK : Election	**FPP**: **LHI**, *percent*	**Australia:** Election	**AV** Count: **LHI**, *percent* First	Final *
Feb 1974	19.9	1983	15.2	11.2
Oct 1974	19.0	1984	11.8	7.9
1979	15.3	1987	13.6	9.8
1983	24.2	1990	17.1	5.0
1987	20.9	1993	14.1	7.4
1992	18.0	1996	18.8	12.6
1997	21.2	1998	20.5	6.4
2001	22.1	2001	18.2	4.9
2005	20.7	2004	15.8	6.6
1974–2005 Mean	20.1	1983–2004 Mean (**First + Final**)	16.1	8.0 (**12.0**)

* AV Final Count: Two-Candidate Preferred (excluding few non-transferable votes: in Australia, valid voting necessitates rank-ordering all AV preferences).

Data Sources: Rallings and Thrasher [19]; Electoral Commission [5]; and Australian Electoral Commission (personal communications, 1988–2005).

Table 4.6: Loosemore-Hanby Index

By Assembly, Electoral System and Election (Year): **UK**, 1995–2005.

Assembly	**Electoral System**	Year	**LHI**, *percent*
House of Commons (UK MPs)	**FPP** (First-Past-the-Post)	2001 2005	22.1 20.7
European Parliament (British MEPs)	**CPL** (Closed Party List: Regional d'Hondt)	1999 2004	14.1 14.7
London Assembly	**FPP** + 44% **CPL** (Party List $V_P\% > 5\%$)	2000 2004	14.8 13.6
National Assembly for Wales	**FPP** + 33% **CPL** (Regional d'Hondt)	1999 2003	11.2 14.1
Scottish Parliament	**FPP** + 43% **CPL** (Regional d'Hondt)	1999 2003	10.5 12.5
Northern Ireland Assembly	**STV** (Six Seats per Constituency)	1998 2003	6.0 to 3.8* 6.4 to 5.4*

* First to Final count (excluding non-transferable votes).

Data Sources: Chief Electoral Officer for Northern Ireland [3]; Electoral Commission [5]; Electoral Office for Northern Ireland [8]; Rallings and Thrasher [19]; **Guardian**, 6 May 2000, 3 May 2003 and 7 May 2005; **Times**, 12 June 2004.

Table 4.7: National Aggregate Loosemore-Hanby Index

By STV District Magnitude, Count and Election:
Irish Republic, 1961–2002.

Election	**District Magnitude** (Seats per STV Constituency): **LHI**, *percent* (First to Final Count*)			
Year (Month)	Total	3	4	5
1961	8.4 to 3.4	9.4 to 4.5	10.7 to 7.1	9.7 to 4.7
1965	3.2 to 2.3	3.2 to 2.0	6.0 to 5.8	4.2 to 2.1
1969	7.1 to 4.5	7.3 to 4.6	7.5 to 4.5	4.3 to 2.0
1973	4.3 to 1.2	4.5 to 2.4	4.6 to 2.6	7.3 to 8.9
1977	7.4 to 4.1	7.3 to 6.0	9.7 to 4.1	8.5 to 1.1
1981	5.8 to 2.4	4.6 to 2.3	10.2 to 2.6	5.3 to 4.0
1982 (Feb)	3.4 to 1.9	2.6 to 2.0	4.4 to 2.8	4.2 to 1.1
1982 (Nov)	4.2 to 1.9	2.6 to 3.8	7.2 to 3.0	4.7 to 3.4
1987	9.9 to 1.3	10.5 to 7.3	10.9 to 2.8	10.1 to 2.2
1989	7.1 to 2.4	6.0 to 3.9	8.9 to 2.5	7.8 to 2.6
1992	8.2 to 3.7	9.8 to 3.6	10.5 to 5.6	8.5 to 3.9
1997	12.9 to 5.1	14.9 to 6.9	16.2 to 6.7	13.2 to 5.7
2002	12.6 to 6.6	15.8 to 10.4	14.2 to 6.3	11.4 to 5.6
1961–2002 Mean (**First + Final**)	7.3 to 3.1 (**5.2**)	7.6 to 4.6 (**6.1**)	9.3 to 4.3 (**6.8**)	7.6 to 3.6 (**5.6**)

* Final Count: Excluding non-transferable votes.

Data source: Dáil Éireann (1962–2003).