Cardinal-weighted pairwise comparison

J. Green-Armytage
jarmyta@antioch-college.edu

James Green-Armytage graduated from Antioch College in April of 2004. He is considering a graduate program for an economics PhD.

1 Introduction

This paper introduces a new voting method named **cardinal-weighted pairwise comparison**, or **cardinal pairwise** for short. It is based on Condorcet's method of pairwise comparison, but in addition to asking voters to rank the candidates in order of preference, this method also asks them to rate the candidates, for example on a scale from 0 to 100. The ordinal ranking information is still used to decide the winner and loser of each pairwise comparison, but the cardinal rating information is used to decide the relative strength of the pairwise victories/defeats, which determines how majority rule cycles are resolved if they occur.

Sections 2 through 4 are primarily concerned with definition, and sections 5 through 7 are primarily concerned with analysis and justification. In sections 2, 3 and 4, I define some key terms, define the cardinal pairwise method, and give an example computation. In section 5, I argue that pairwise methods in general are superior to other voting methods when the goal is majority rule. In sections 6 and 7, I discuss the advantages of cardinal pairwise over other pairwise methods, which are as follows: First, it takes into account the relative priority of each pairwise preference to each voter. Second, it may greatly reduce the vulnerability to strategic manipulation that is troublesome for pairwise methods.

2 Preliminary definitions

Pairwise comparison, pairwise defeat, pairwise tie: A pairwise comparison uses ranked ballots to simulate head-to-head contests between different candidates. Given two candidates A and B, there is a pairwise defeat of B by A if and only if A is ranked above B on more ballots than B is ranked above A. If the number of A>B ballots is equal to the number of B>A ballots, then there is a pairwise tie between A and B.

> and = symbols: I use these in two slightly different ways. For example, "A>B" can mean that an individual voter or a specific set of voters ranks A above B, and it can also mean that A has a pairwise victory over B. "A=B" can signify an equal ranking of A and B, or a pairwise tie between A and B. The meaning will be made clear by the context.

Condorcet winner, Condorcet-efficiency, Condorcet criterion: A Condorcet winner, also called a 'dominant candidate,' is a candidate that wins all of its pairwise comparisons. If a voting method always elects a Condorcet winner when one exists, the method is Condorcet-efficient, and passes the Condorcet criterion.

Strong Condorcet winner: A Condorcet winner whose pairwise victories are each supported by more than one half of the ballots.

Majority rule cycle: A circular series of pairwise defeats (e.g. A beats B, B beats C, C beats A) that leaves no single candidate unbeaten.

Condorcet completion method: A voting method that chooses the Condorcet winner when one exists, and is also decisive when there is no Condorcet winner. The following four methods (minimax, ranked pairs, river, and beatpath) are Condorcet completion methods.

Minimax method: The winner is a candidate whose strongest pairwise loss (if any) is the least strong compared to other candidates' strongest losses. Equivalent to a method that drops the weakest pairwise defeat until one candidate is undefeated.

Ranked pairs method: Defeats are considered in descending order of strength. They are locked in place unless they make a cycle with already-locked defeats, in which case they are skipped. The winner will be a candidate who is undefeated after all the defeats have been considered. See Tideman [11].

River method: Similar to ranked pairs, except that it does not lock more than one defeat against the same candidate; once the first has been locked, any others are skipped. See Heitzig [3].

Beatpath method: A beatpath is a series of pairwise defeats that form a path from one candidate to another. For example, if A beats B, and B beats C, then there is a beatpath from A to C. The strength of a beatpath is defined as the strength of its weakest component defeat. If the strongest beatpath from X to Y is stronger than the strongest beatpath from Y to X, then X has a beatpath win over Y. The winner of the beatpath method will be a candidate such that no other candidate has a beatpath win against it. See Schulze [8].

Ordinal pairwise: A shorthand term that I will use to refer to versions of the minimax, ranked pairs, river, and beatpath methods that only use ordinal rankings, and measure defeat strength in terms of a sheer number of votes, whether the number of votes in agreement with a defeat, or the margin between the number of votes in agreement and the number of votes in disagreement.

Minimal dominant set: The smallest set of candidates such that every candidate within the set has a pairwise victory over every candidate outside the set. See Schwartz [10]. The ranked pairs, river, and beatpath methods always choose from the minimal dominant set, whereas the minimax method does not.

Resolvability: A voting method is resolvable if the probability that a random solution will be needed to produce a winner approaches zero as the number of voters approaches infinity.

Mutual majority criterion: If there is a single majority of the voters who rank every candidate in a set S_1 over every candidate outside S_1, then the winner should always be a member of S_1.

3 Definition of the cardinal-weighted pairwise comparison method

3.1 Ballot

1. Voters rank the candidates. Equal rankings are allowed.
2. Voters rate the candidates, e.g. on a scale from 0 to 100. Equal ratings are allowed. If you give one candidate a higher rating than another, then you must also give the higher-rated candidate a higher ranking.

3.2 Tally

1. Determine the *direction* of the pairwise defeats by using the *rankings* for a standard pairwise comparison tally.
2. Determine the *strength* of the pairwise defeats by finding the weighted magnitude as follows. Suppose that candidate A pairwise beats candidate B, and we want to know the strength of the defeat. For each voter who ranks A over B, and *only* for voters who rank A over B, subtract their rating of B from their rating of A, to get the rating differential. The sum of these individual winning rating differentials is the total weighted magnitude of the defeat. (Note that voters who rank B over A do not contribute to the weighted magnitude of the A>B defeat; hence it is never negative.)
3. Now that the direction of the pairwise defeats have been determined (in step 1) and the strength of the defeats have been determined (in step 2), you can choose from a variety of Condorcet completion methods to determine the winner. I recommend the ranked pairs, beatpath, and river methods.

3.3 Optional, additional provisions

These additional provisions are not an essential part of the cardinal-weighted pairwise method, but they may prove helpful.

1. **Maximizing in scale provision**: [1] Once a minimal dominant set has been established by the pairwise tally in step 2, it may be a good idea to max-

imize the voters' rating differentials in scale between the candidates in the set. That is, to change the ratings on each ballot so that the highest-rated minimal dominant set candidate is at 100, the lowest-rated minimal dominant set candidate is at 0, and the rating differentials between the minimal dominant set candidates retain their original ratios. (For example, 50,20,10 would become 100,25,0.) The benefit of this provision is that voters will have equal ballot weight with regard to the resolution of the majority rule cycle in particular. Therefore, voters will not have an incentive against investing priority in preference gaps that are relatively unlikely to fall within the minimal dominant set.

2. **Blank rating option**: This allows voters to give one or more candidates a blank rating, such that if I give some candidate a blank rating, my ballot will still affect the direction of pairwise defeats concerning that candidate, but it will not add to the weighted magnitude of such defeats.

   Another possible way to deal with candidates that voters leave unrated is to determine their ratings using a default formula. For example, a candidate ranked in first place could be given a default rating of 100, a candidate ranked in last place could be given a default rating of 0, and remaining default ratings could be spaced evenly within the constraints imposed by surrounding ratings.

4 An example computation

The notation in the first line below is used to indicate that 26 voters rank the candidates in the order Right > Left$_B$ > Left$_A$, and assign the three candidates ratings of 100, 10, and 0, respectively.

4.1 Example

26: Right > Left$_B$ > Left$_A$ (100,10,0)
22: Right > Left$_A$ > Left$_B$ (100,10,0)
26: Left$_B$ > Left$_A$ > Right (100,90,0)
1: Left$_B$ > Right > Left$_A$ (100,50,0)
21: Left$_A$ > Left$_B$ > Right (100,90,0)
4: Left$_A$ > Right > Left$_B$ (100,50,0)
Direction of defeats (using ranking information):
Right > Left$_B$: 52-48
Left$_A$ > Right: 51-49
Left$_B$ > Left$_A$: 53-47

Weighted magnitude of defeats (using rating information): Right > Left$_B$:
$(26\times(100-10))+(22\times(100-0))+(4\times(50-0)) = 4740$

Left$_B$ > Left$_A$:
$(26\times(10-0))+(26\times(100-90))+(1\times(100-0)) = 620$

Left$_A$>$Right$:
$(26\times(90-0))+(21\times(100-0))+(4\times(100-50)) = 4640$

Completion by cardinal-weighted pairwise with **ranked pairs** or **river**: Consider the defeats in the order of descending weighted magnitude.
4740: Right > Left$_B$ keep
4640: Left$_A$ > Right keep
620: Left$_B$ > Left$_A$ skip (would cause a cycle, Right>Left$_B$>Left$_A$>Right)
Kept defeats produce ordering Left$_A$>Right>Left$_B$; *Left$_A$ wins.*

Completion by cardinal-weighted pairwise with **beatpath**: The strength of a beatpath is defined by the defeat along that path with the lowest weighted magnitude.
beatpath Right → Left$_B$: 4740
beatpath Left$_B$ → Right: 620
beatpath Left$_A$ → Right: 4640
beatpath Right → Left$_A$: 620
beatpath Left$_A$ → Left$_B$: 4640
beatpath Left$_B$ → Left$_A$: 620
Complete ordering is Left$_A$>Right>Left$_B$; *Left$_A$ wins.*

5 Why majoritarian election methods should be Condorcet-efficient

The Condorcet criterion (along with the minimal dominant set, which is a generalization of the same principle) seems to be the most authentic definition of majority rule that is available to us. If there is one candidate who is preferred by some majority over every other candidate individually, it seems inappropriate to call anyone else a majority winner. For example, if candidate A is a Condorcet winner, and a non-Condorcet-efficient method elects candidate B, a majority will prefer A to B. If there was an election just between these two candidates, A should be expected to win that election.

Condorcet efficiency has important practical benefits. First, Condorcet-efficient methods tend toward the political center, which should promote compromise rather than polarization. Second, when a strong Condorcet winner exists with respect to voters' sincere preferences, and another method chooses someone else, the result is unstable in that a majority could have achieved a mutually preferable result if some of them had voted differently.

Condorcet-efficient methods minimize the incentive for the **compromising strategy**, which is insincerely ranking an option higher in order to decrease the probability that a less-preferred option will win. For example, if my sincere preferences are R>S>T, a compromising strategy would be to vote S>R>T or R=S>T, raising S's ranking in order to decrease T's chances of winning. (The drawback is that this often decreases R's chances of winning as well.) All resolvable voting methods that satisfy the mutual majority criterion have a compromising incentive when there is a majority rule cycle. But unlike other methods, such as single-winner STV, voters in Condorcet-efficient methods never have an incentive to use the compromising strategy when there is a Condorcet winner [9]. This is an important property because, in the absence of a majority rule cycle, it allows me to vote my R>S preference without worrying that it will undermine my S>T preference. This is a more complete way of curtailing the "lesser of two evils" problem, that is, decreasing the extent to which voters have to worry about earlier choices drawing support away from later choices. Thus, Condorcet-efficient methods allow more candidates to participate on an equal basis, which should lead in turn to substantially higher levels of responsiveness and accountability.

6 Preference priority and defeat strength

Most Condorcet-efficient methods that have been proposed so far limit voter input to ordinal rankings. Hence, voters can express preferences between candidates, but they cannot express the relative priority of their preferences. If I worship my first three choices, but detest my fourth and fifth choices, I cannot express this on my ballot, and it is not taken into account when the winner is decided.

Ordinal pairwise methods measure defeat strength in terms of a sheer number of ballots. The cardinal pairwise method extends the sensitivity of the process by factoring in a measure of how much priority the voters assign to each ranking. The goal is that the weakest defeat in a majority rule cycle should be the one that has the lowest overall combination of these two factors: 1) the number of voters in agreement with the defeat; 2) the relative priority of the defeat to those voters who agree with it.

It seems almost axiomatic that, when faced with a majority rule cycle, one should drop the defeat(s) in the cycle that are of least importance to the voters. The remaining question is how to define the priority of each defeat to each voter, and how to aggregate these individual priorities. The answer that cardinal pairwise gives to this question is relatively simple. For those who agree with a defeat, we look at the rating differential they express between the two candidates being compared. Then we take the sum of these winning rating differentials to find the overall strength of the defeat.

The idea is that the voters will rate the candidates such that the rating differential between each pair of candidates will reflect the relative priority of their preference between those candidates. The fact that each voter is constrained to the same range of ratings (e.g. 0 to 100) assures that everyone has essentially the same voting "power." The point here is not to do interpersonal comparison of utilities, but rather to allow voters to prioritize their own preferences relative to one another, using a fluid and simple high-resolution scale.

When learning the cardinal pairwise method, one may wonder why it only looks at the rating differentials of those who agree with a particular defeat, rather than subtracting the losing rating differentials from the winning rating differentials. To begin with, I will say that I am more interested in dropping the defeats that are of least importance to the voters overall, rather than the defeats that are the closest in terms of the strength of preference on either side. That is, if there is one pairwise comparison that voters on both sides consider to be a very high priority, I think that it is especially important not to reverse this defeat. Such high-priority defeats should be regarded as crucial within the election, and the cardinal aspect of the method should be used to defend them rather than to undermine them.

In this way, looking at only the winning rating differentials greatly improves the *stability* of the cardinal pairwise method. Because the defeats that voters place the highest priority on are the most difficult to reverse, the cardinal pairwise method is unusually resistant to strategic manipulation. This point will be explored in greater detail in the next section.

7 Strategic manipulation

Although Condorcet-efficient methods minimize the incentive for use of the compromising strategy, they are vulnerable to the **burying strategy**. This strategy entails insincerely ranking an option lower in order to increase the probability that a more-preferred option will win. For example, if my sincere preferences are R>S>T, a burying strategy would be to vote R>T>S or R>S=T, lowering S's ranking in order to increase R's chances of winning. (The drawback is that this often increases T's chances of winning as well.)

Imagine that with respect to voters' sincere preferences in a three-candidate election, A pairwise beats B and C, while B pairwise beats C. A is a sincere Condorcet winner, but it is often possible for supporters of candidate B to gain an advantage by burying A under C, that is, by voting B>C>A instead of B>A>C. This can create an insincere C>A defeat, which can cause a majority rule cycle such that the A>B defeat is the weakest of the three, so that B wins. In this way, it is often possible to overrule a genuine defeat with a fake defeat.

The burying strategy may have the potential to cause substantial trouble in elections that use a Condorcet-efficient method. Some have cited this as a reason not to adopt Condorcet-efficient methods. (Monroe [5]; Richie and Bouricus [6]) Unfortunately, Condorcet-efficient methods cannot be completely invulnerable to the burying strategy, which follows from the fact that Condorcet-efficiency is incompatible with the later-no-help criterion [12]. However, cardinal pairwise may be able to make this vulnerability much less severe.

There are many reasons to think that cardinal pairwise will be more resistant to strategy than most other Condorcet-efficient methods. First, it should tend to prevent the most flagrant strategic incursions. Second, it should tend to balance strategic incentive against strategic ability, so that those who are most interested in changing the result via strategic incursion tend to be those who are least able to do so. Third, it should minimize strategic barriers against the entry of new candidates. Fourth, it should create the possibility of more-stable counterstrategies than those that are available in ordinal pairwise.

7.1 Flagrant strategic incursions

I define a flagrant strategic incursion as one that causes a very high-priority defeat to be overruled by a false defeat. Take example 7.1 below. Sincere votes:

46: A>B>C (100,10,0)
44: B>A>C (100,10,0)
5: C>A>B (100,50,0)
5: C>B>A (100,50,0)

A is a Condorcet winner. Clearly, the primary contest is between A and B, as C is the last choice of 90% of the voters. However, using ordinal pairwise, the B>A>C voters can change the winner to B by voting B>C>A. This is a very flagrant incursion.

In cardinal pairwise, however, this particular type of flagrant incursion does not work. The weighted magnitude of the C>A defeat is 4490, and no defeat with a magnitude greater than $3333^{1}/_{3}$ can be dropped as a result of a three candidate cycle (assuming 100 voters and a 0-100 rating scale).

With larger cycles (four candidates and above, e.g. A>B>C>D>A), the $3333^{1}/_{3}$ limit does not apply, but overruling a high-magnitude defeat is still very difficult. Let's say that there is a candidate B, who is pairwise-beaten by a candidate A. In order for B to win, there must be a chain of defeats from B to A (e.g. B>C>D>A), such that every defeat along that chain has a weighted magnitude that is at least equal to the A>B defeat. The minority who prefer B to A will have a limited amount of weight to distribute along the B>C>D>A chain. A given point of weight can count towards two defeats in this four-candidate chain (e.g. the one-point gap in the vote B>D>C>A (1,1,0,0) counts towards the B>C and D>A defeats), but it cannot count towards more than two.

Cardinal pairwise, unlike ordinal pairwise, does not allow a voter to apply the maximum weight to all of their pairwise preferences. This scarcity of weight produces excellent anti-strategic effects, by placing a limit on the extent to which a strategizing group of voters can build up the weight of multiple pairwise defeats at the same time in order to manipulate the overall result.

In general, flagrant incursions are much less likely to be successful in cardinal pairwise than in ordinal pairwise, because the difficulty of overruling an A>B defeat increases as more voters assign a higher priority to the A>B defeat. I hope that my definition of a flagrant incursion can be seen to have value, and that it can be agreed upon that relatively high-priority defeats should be harder to overrule. Consider that when a defeat of A over B is given a very high priority, we can generally expect B to be very *different* from A (in the eyes of the voters), relative to differences with the other candidates in the election. In order to quantify this difference, we can look at both the average A>B rating differential and the average B>A rating differential for individual voters.

I think it is crucial that we make it as difficult as possible for strategic voters to alter an election result in such a way that the actual winner is considered by the voters to be extremely different from all of the members of the sincere minimal dominant set. Consider how seriously it would undermine the legitimacy of the voting system, if it was found that partisan supporters had pulled off a successful burying strategy which won the election for a candidate who was the ideological polar opposite of the sincere Condorcet winner. Ordinal pairwise unfortunately cannot offer much protection against this disturbing possibility, but cardinal pairwise can.

7.2 Strategic incentive and strategic ability

There are impossibility theorems that show that strategic manipulation cannot be completely avoided in any reasonable election method (Gibbard [2]; Satterthwaite [7]; Hylland [4]), but I'm not aware of a theorem that says that we can't find a method that distributes strategic ability in roughly inverse proportion to strategic incentive.

Let's assume that the intensity of difference that a voter perceives between two candidates tends to be largely independent of their ranking of those candidates, and that the average rating differentials on either side of a defeat will tend to be strongly correlated with one another.

Let's say that there is a candidate A who pairwise beats candidate B. If the incentive for the B>A voters to help B by burying A is particularly strong–that is, if they assign a very high priority to their B>A ranking–then we can expect the A>B voters to assign a high priority to their A>B ranking as well, which will make the A>B defeat very hard to overrule. So, a group of voters' ability to achieve a successful burying strategy generally tends to be smaller in cases where that group has a larger incentive to engage in that strategy.

Conversely, if A and B are considered to be more similar candidates, such that there are low average rating differentials on both sides of the defeat, then it may be more feasible for the B>A voters to help B by burying A, but they would have less to gain by doing so, and more to lose should the strategy backfire.

7.3 Minimizing strategic barriers to candidate entry

In example 4.1 above, Left_B and Left_A can be considered to be relatively similar candidates, in that there is a low average rating differential placed on the comparison between them, going in both directions. If only Left_A and Right were candidates, Left_A would probably win, since he has a pairwise win over Right. In cardinal pairwise, the entry of Left_B does not change this result. However, the winner changes to Right in ordinal pairwise, which defines Right's 49-51 pairwise loss as the weakest in the cycle. In general, it is much harder in cardinal pairwise for the entry of a new, non-winning candidate to do harm to a similar candidate. The reason for this is that if the new candidate beats the similar candidate, but does not win, this defeat will be relatively weak, and hence likely to be overruled in the event of a cycle.

In ordinal pairwise, a voter who would otherwise support a potentially-entering candidate might have some anxiety that this candidate could hurt a similar candidate whom that voter also supports. Because the potentially-entering candidate's support base may feel ambivalent about his presence in the race, entry of the candidate may not occur. Thus, the method retains a certain strategic barrier to entry of new candidates. Cardinal pairwise minimizes this barrier to entry, in that the entry of a new candidate is extremely unlikely to affect the result in opposition to the will of his would-be supporters.

7.4 Stable counterstrategies

If several voters try to coordinate a strategic incursion, and other voters learn about this and consider it to be undesirable, they may attempt to coordinate a counterstrategy, in order to make the initial strategy unsuccessful. One hopes that counterstrategy will rarely or never be needed, but it is nevertheless to the credit of cardinal pairwise that it provides for somewhat more-stable counterstrategies than ordinal pairwise. Actually, this may be important in preventing strategic incursion from achieving a critical mass in the first place.

Example 7.2: Some votes are strategically altered

28: A>B>C (100,60,0)
23: C>A>B (100,40,0)
17: B>A>C (100,60,0)
22: C>B>A (100,40,0)

10: B>C>A (100,100,0) these 10 votes are strategically altered from a sincere ordering of B>A>C

Pairwise comparisons, followed by weighted magnitudes:
A > B: 51-49   C > A: 55-45   B > C: 55-45
A > B: 2040   C > A: 4580   B > C: 3380
Candidate A was a sincere Condorcet winner, but B wins instead using both ordinal and cardinal pairwise, as a result of the B>A>C voters' burying strategy.

There are two basic counterstrategy replies to the burying strategy: the compromising counterstrategy, and the deterrent/burying counterstrategy.

In ordinal pairwise, the **compromising counterstrategy** would entail the C>A>B voters weakening or reversing the defeat against A by voting C=A>B. In cardinal pairwise, a similar effect could be gained by voting C>A>B (100,100,0). Both counterstrategies can return the victory to candidate A. The cardinal pairwise counterstrategy is more stable than the ordinal pairwise counterstrategy, in that it does not risk a change in the winner of the A-C pairwise comparison. This makes it a less perilous choice for the C>A>B voters.

The **deterrent/burying counterstrategy** would entail the A>B>C voters weakening or reversing B's defeat of C, such that the B>A>C voters' burying of A could only backfire by electing C. In ordinal pairwise, this would require some A>B>C voters to equalize or reverse their B>C preference, thus voting A>B=C or A>C>B. In cardinal pairwise, it is possible for the A>B>C voters to get a similar deterrent effect by voting A>B>C (100,0,0).

With the deterrent/burying counterstrategy in general, the counterstrategizers are unlikely to know for sure whether the original strategizers will carry out their incursion or not, until the votes have already been cast. Therefore it is important to have an effective counterstrategy that they can use without severely destabilizing the result, in case the original strategy is not carried out and the counterstrategy punishment is undeserved. In this respect, the cardinal pairwise version of the counterstrategy is preferable, in that it does not alter the direction of any pairwise defeats, and therefore will not interfere with the identification of a Condorcet winner.

Of course, the existence of more-stable counterstrategies in cardinal pairwise does not mean that strategy will never be a problem. However, it suggests to me that the threat of a strategic incursion, should it arise, is less likely to spiral out of control.

8 Conclusion

I believe that voting methods aiming for majority rule should be Condorcet-efficient, and that Condorcet-efficient methods should be improved in two ways. One, they should take the relative priority of voters' pairwise preferences into account; two, they should be more resistant to the burying strategy. I find it serendipitous that the same principle can achieve both benefits simultaneously.

I find both of these potential improvements quite significant, but perhaps the strategic issue is the more pressing of the two, as I suspect that the burying strategy could prove to be a serious problem for Condorcet-efficient methods in contentious elections. It is important to have a method that, in addition to recognizing a Condorcet winner when one is clearly expressed, works to protect sincere Condorcet winners from being obscured by strategic incursion. I believe that cardinal-weighted pairwise accomplishes this to an unusual degree.

So, I do not intend cardinal-weighted pairwise as a frivolous academic exercise or a mathematical curiosity. I intend it as a realistic proposal, and one that I sincerely prefer over other existing proposals. I recognize that it adds an extra layer of complexity, but I feel that the benefits of more-meaningful cyclic resolution and reduced strategic vulnerability far outweigh the cost.

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