

# Voting matters

*for the technical issues of STV*

published by

The McDougall Trust

Issue 18

June 2004

**About the McDougall Trust (reg. charity no. 212151)**

The McDougall Trust is a charitable trust formed in 1948. The charity's purposes as stated in its governing scheme of 1959 are to advance knowledge of and encourage the study of and research into:

- political or economic science and functions of government and the services provided to the community by public and voluntary organisations; and
- methods of election of and the selection and government of representative organisations whether national, civic, commercial, industrial or social.

The Trust's work includes the maintenance and development of the Lakeman Library for Electoral Studies, a unique research resource, the production and publication of *Representation: The Journal of Representative Democracy*, and, of course, this publication **Voting matters**, that examines the technical issues of the single transferable vote and related electoral systems.

For further information on the Trust, please contact:

The Secretary,  
McDougall Trust,  
6 Chancel Street,  
London SE1 0UX, UK.  
Telephone: +44 (0)20 7620 1080  
Facsimile: +44 (0)20 7928 1528  
Email: [admin@mcdougall.org.uk](mailto:admin@mcdougall.org.uk)  
Web: [www.mcdougall.org.uk](http://www.mcdougall.org.uk)

For further information on this publication, please contact B A Wichmann, the Editor at the above address or by email at: [Brian.Wichmann@bcs.org.uk](mailto:Brian.Wichmann@bcs.org.uk)



## Editorial

I would strongly recommend to all readers that the Interim Report of the Commission on Electronic Voting, issued by the Irish Government, is studied closely. This Commission, was formed on the 1st March and required to report by 1st May, on the suitability of the system chosen for use in elections in Ireland. They recommended that the chosen system should *not* be used for the local/European elections to be held on 11th June. The Commission's Report can be downloaded at: [http://www.cev.ie/htm/report/download\\_report.htm](http://www.cev.ie/htm/report/download_report.htm)

To avoid any confusion, I need to declare an interest in this report, since I worked with Joe Wadsworth of Electoral Reform Services in testing the counting engine of the official software. Our work was not finished until the end of March, which was only 5 weeks before the Commission reported.

Some aspects of their report are of particular interest here:

- The desirability of removing random selection in the counting process;
- Problems associated with full disclosure of the ballot data (discussed further in this issue);
- Some shortcomings with regard to secrecy;
- The need for a Voter Verifiable Audit Trial.

A Voter Verifiable Audit Trial might work by having a printer attached to the electronic voting machine which printed out the filled-in paper after it had been recorded electronically. The voter would then check this, and place the paper in a conventional ballot box. Hence the ballot box papers can be used as a (manual) check against the computer count.

Technically, such a scheme has a number of problems. Firstly, printers are less reliable than a purely electronic device; should the printer jam, the election officials might inadvertently see a ballot paper. Secondly, the conventional record would presumably be used for a recount; however, a manual recount is likely to be less reliable than the initial electronic count. The process whereby the printed papers are used needs to be very carefully considered.

There is no doubt that the undertaking of a manual count is one that the public feels gives confidence in the democratic process. What, therefore, needs to be done to gain the same confidence in an electronic count? The

Irish report gives some insight into this important issue. Is it necessary to have a Voter Verifiable Audit Trial, in spite of the problems noted above? Since the Irish Government is still planning to use electronic voting, we will soon be able to see how these issues are being addressed.

Returning to *Voting matters*, there are 6 papers in this issue:

- I. D. Hill: What is meant by 'monotonic'? What is meant by 'AV'?
- M. Schulze: Free riding.
- I. D. Hill: An odd feature in a real STV election.
- I. D. Hill: Full disclosure of data.
- B. A. Wichmann: A note on the use of preferences.
- J. C. O'Neill: Tie-Breaking with the Single Transferable Vote.

David Hill highlights the problem of the meaning of terms and even abbreviations. As Editor, I am always concerned about this, since the terminology in common use varies substantially, especially now that papers are authored from outside the UK/Ireland.

Markus Schulze raises the interesting and important question of the extent to which strategic voting is used in STV elections. Two forms of strategic voting are analysed, which in one case, can be identified from US ballot data in which voters can *write-in* a candidate. Fortunately for STV, the analysis gives no evidence of strategic voting in the analysable case.

The next three articles are all about the use of preferences. David Hill first provides an example in which a single paper with a large number of preferences has a crucial effect. His subsequent papers respond to an earlier *Voting matters* paper on full disclosure. In my own article, I consider the actual use made of the preferences specified by the voter, and how this information could be altered to avoid any undesirable consequences of full disclosure.

In the final article, Jeff O'Neill analyses the various ways in which ties are broken which results in a proposal to change the tie-breaking logic in the current Electoral Reform Society rules.

*Readers are reminded that views expressed in **Voting matters** by contributors do not necessarily reflect those of the McDougall Trust or its trustees.*



# What is meant by ‘monotonic’? What is meant by ‘AV’?

I. D. Hill

No email available.

It is said that, during the 1939-1945 war, Winston Churchill and President Roosevelt had a disagreement when Churchill wished to table a document and Roosevelt did not wish it to be tabled. It turned out that they both wanted the same thing: that to the British, tabling a document means putting it on the table for discussion; whereas to the Americans, it means putting it in a drawer and forgetting it. Such confusion, caused by language difficulties, can be serious.

## 1 Monotonic

Schulze [1] explains a method for single seat elections that finds the Condorcet winner if there is one, and has a strategy for choosing a winner where there is a Condorcet paradox. He claims that the method is “monotonic and clone-independent”.

The main purpose of this note is to warn others who may have been misled, as I was myself at first, by that claim. The trouble lies in definitions, because I am told that his usage of ‘monotonic’ is as normally used in the social choice literature, but it is a much narrower definition than is often taken as the meaning in electoral reform literature.

He gives an example where his method certainly violates the condition that Woodall [2] calls mono-add-top: “A candidate  $x$  should not be harmed if further ballots are added that have  $x$  top (and are otherwise arbitrary)”, but Schulze is only claiming to meet mono-raise: “A candidate  $x$  should not be harmed if  $x$  is raised on some ballots without changing the orders of the other candidates”.

I am not seeking to cast any blame. If that usage of the word is widely employed, he is fully entitled to follow it, but a clash of definitions may cause misunderstanding if we do not take great care.

## 2 AV

Brams and Fishburn [3] give an example of the use of a system called Approval Voting, and they use AV as an abbreviation for it. In this country AV has been used for many years to mean the system called Alternative Vote.

Approval Voting is a system in which a voter uses X-voting for as many candidates as desired, even when there is only one seat to fill. The winner is the one who gets the most Xs. Alternative Vote is what STV reduces to in the single-seat case, voting by preference number, with eliminative counting.

It is not my purpose in this note to examine the relative merits, or lack of merits, of these two systems, but only to warn that they are very different, and that the name AV is, unfortunately, being used for both of them. Again, this may cause misunderstanding if we do not take great care.

## 3 References

- [1] Schulze M. A new monotonic and clone-independent single-winner election method. *Voting matters*, 17, 9-19. 2003.
- [2] Woodall D.R. Properties of preferential election rules. *Voting matters*, 3, 8-15. 1994.
- [3] Brams S.J. and Fishburn P.C. A nail-biting election. *Social Choice and Welfare*, 18, 409-414. 2001.

A special thanks to David Hill for checking this issue.

# Free riding

Markus Schulze  
markus.schulze@alumni.tu-berlin.de

## 1 Introduction

The fact that more and more communities that use proportional representation by the single transferable vote (STV) change from manual count to computer count gives us today the possibility to check hypotheses that have been made in the past about possible voting behaviours. In this paper, I use the ballot data of the 1999 and the 2001 City Council elections and School Committee elections in Cambridge, Massachusetts, to estimate the number of voters who use a voting behaviour that has been predicted e.g. by Woodall [1] and Tideman [2].

## 2 Woodall Free Riding

Woodall free riding is a useful strategy only for those STV methods where votes of eliminated candidates cannot be transferred to already elected candidates and therefore jump directly to the next highest ranked *hopeful* (i.e. neither yet elected nor yet eliminated) candidate. A *Woodall free rider* is a voter who gives his first preference to a candidate who is believed by this voter to be eliminated early in the count even with this voter's first preference. With this strategy this voter assures that he does not waste his vote for a candidate who is elected already during the transfer of the initial surpluses.

Woodall writes [1]:

“The biggest anomaly is caused by the decision, always made, not to transfer votes to candidates who have already reached the quota of votes necessary for election. This means that the way in which a given voter's vote will be assigned may depend on the order in which candidates are declared elected

or eliminated during the counting, and it can lead to the following form of tactical voting by those who understand the system. If it is possible to identify a candidate *W* who is sure to be eliminated early (say, the Cambridge University Raving Loony Party candidate), then a voter can increase the effect of his genuine second choice by putting *W* first. For example, if two voters both want *A* as first choice and *B* as second, and *A* happens to be declared elected on the first count, then the voter who lists his choices as ‘*A B...*’ will have (say) one third of his vote transferred to *B*, whereas the one who lists his choices as ‘*W A B...*’ will have all of his vote transferred to *B*, since *A* will already have been declared elected by the time *W* is eliminated. Since one aim of an electoral system should be to discourage tactical voting, this seems to me to be a serious drawback.”

However, Woodall free riding can be prevented by restarting the STV count with the remaining candidates whenever a candidate has been eliminated. Actually, the Meek method [3] and the Warren method [4] do this. Therefore, Woodall [1] and Tideman [2] suggest that one of these methods should be used.

A good test for Woodall free riding is an STV election with *write-in options* (i.e. with the possibility for the voters to vote for any person by writing this person's name on the ballot). The City Council and the School Committee of Cambridge, Massachusetts, are elected by an STV method that is vulnerable to Woodall free riding and that has write-in options. In the elections to the 9 seats of the City Council, the voter can vote for up to 9 write-ins. In the elections to the 6 seats of the School Committee, the voter can vote for up to 6 write-ins. Here the optimal Woodall free riding strategy is to give one's first preference to a completely unimportant write-in.

|   | CC 1999 | SC 1999 | CC 2001 | SC 2001 |
|---|---------|---------|---------|---------|
| 1 | 18,613  | 17,796  | 17,125  | 16,488  |
| 2 | 28      | 26      | 30      | 51      |
| 3 | 9       | 5       | 12      | 32      |
| 4 | 0       | 4       | 0       | 2       |
| 5 | 19      | 17      | 18      | 17      |

Table 2.1: Potential write-in Woodall free riders in the 1999 and the 2001 elections to the City Council and the School Committee of Cambridge, Massachusetts

In table 2, row “1” contains the numbers of voters in the 1999 City Council elections (column “CC 1999”), in the 1999 School Committee elections (column “SC 1999”), in the 2001 City Council elections (column “CC 2001”), and in the 2001 School Committee elections (column “SC 2001”) in Cambridge, Massachusetts. Row “2” contains the numbers of voters who cast a first preference for a write-in. Row “3” contains the numbers of voters who have to be subtracted from row “2” because they cast preferences only for write-ins and who are therefore obviously not Woodall free riders. Furthermore, those voters who do not cast at least a valid second and a valid third preference have to be subtracted (row “4”) because these voters cannot be Woodall free riders. Therefore, row “5” contains the numbers of voters who could be write-in Woodall free riders.

In all four elections, the number of voters who could be write-in Woodall free riders is about 0.1%. When we investigate these voters in greater detail we observe: Of the 19 potential write-in Woodall free riders in the 1999 City Council elections, only 2 cast a second preference for Galluccio. Of the 17 potential write-in Woodall free riders in the 1999 School Committee elections, only 2 cast a second preference for Turkel. Of the 18 potential write-in Woodall free riders in the 2001 City Council elections, only 5 cast a second preference for Galluccio, 2 for Davis, and one for Murphy. Of the 17 potential write-in Woodall free riders in the 2001 School Committee elections, only 4 cast a second preference for Turkel, one for Fantini, and none for Grassi. Therefore, also these voters seem to be not Woodall free riders because otherwise super-proportionally many of these voters would have cast a second preference for a candidate who reached the quota before candidates had to be eliminated. See table 2.2.

Suppose  $V$  is the number of voters. Suppose  $V_1(A)$  is the number of voters who cast a valid first preference

for candidate A. Suppose  $V_2(A)$  is the number of voters who cast a valid first preference for candidate A and at least also a valid second preference. Suppose  $V(A,B)$  is the number of voters who cast a valid first preference for candidate A, a valid second preference for candidate B, and at least also a valid third preference.

Woodall free riding is a useful strategy only when one has at least a sincere first and a sincere second preference. A given voter can be a Woodall free rider only when he casts at least a valid first, a valid second, and a valid third preference. When a given voter whose sincere first preference is candidate B uses Woodall free riding then  $V_2(B)$  decreases and for some other candidate A, who is eliminated early in the count,  $V(A,B)$  increases. Therefore, another good test for Woodall free riding is to calculate  $V(A,B)$  for each pair of candidates. If (1)  $V(A,B)/V_1(A)$  is large compared to  $V_2(B)/V$  and (2)  $V(A,B)/V_1(A)$  decreases with increasing  $V_1(A)$  for those pairs of candidates where candidate A is eliminated early in the count and candidate B is elected before candidates have to be eliminated then this is evidence that voters use Woodall free riding.

Table 2.2 contains  $V_2(B)/V$  for each candidate B who is elected before candidates have to be eliminated. Tables 2.3 to 2.6 contain  $V(A,B)$  for each pair of candidates A and B where candidate B is elected before candidates have to be eliminated. Column “ $V_1(A)$ ” contains the numbers of voters who cast a valid first preference for the candidate in column “candidate A”. The column “Galluccio” (resp. “Turkel”, resp. “Davis”, etc.) contains the numbers of voters of column “ $V_1(A)$ ” who cast a valid second preference for Galluccio (resp. Turkel, resp. Davis, etc.) and cast at least also a valid third preference.

In tables 2.3 to 2.6,  $V(A,B)/V_1(A)$  rather increases than decreases with increasing  $V_1(A)$ . Also the prediction that  $V(A,B)/V_1(A)$  is large compared to  $V_2(B)/V$  is not fulfilled. This is surprising because in so far as Woodall free riding certainly is a useful strategy one would expect that at least some voters use this strategy. A possible explanation why voters do not use Woodall free riding is that they fear that when too many voters give their first preference to candidate A because they believe that he is eliminated early in the count then it could happen that candidate A gets so many votes that he is elected [2, 5, 6]. But this can only explain why  $V(A,B)/V_1(A)$  does not decrease so fast with increasing  $V_1(A)$ ; this cannot explain why  $V(A,B)/V_1(A)$  increases with increasing  $V_1(A)$ . A possible explanation why  $V(A,B)/V_1(A)$  increases with increasing  $V_1(A)$  is

that voters are confronted with two problems:

1. It is a useful strategy not to waste one's vote by voting for a candidate B who is elected even without one's vote. However, when too many voters use Woodall free riding and cast a first preference for candidate A because they believe that he is eliminated early in the count even with one's vote then it could happen that candidate A gets so many votes that he is elected.
2. It is a useful strategy not to vote for a candidate A who is believed to be eliminated with a great probability even with one's vote, because otherwise there is the danger that there are not acceptable candidates anymore to whom this voter could transfer his vote when candidate A is eliminated.

Because of problem 2 only those voters who cannot identify themselves with any of the stronger candidates vote for candidates who are believed to be eliminated with a great probability; therefore,  $V(A,B)/V_1(A)$  is low for low  $V_1(A)$  for those candidates B who are elected before candidates have to be eliminated; therefore,  $V(A,B)/V_1(A)$  rather increases than decreases with increasing  $V_1(A)$ .

### 3 Hylland Free Riding

Problem 1 can be circumvented by using Hylland free riding instead of Woodall free riding. Hylland writes [7]:

“Both for groups and for individual voters it could be advantageous not to vote for a candidate who is considered certain of winning election, even if that candidate is one's first choice. Suppose that my true first and second choices are A and B, I am sure A will get many more first preferences than needed for election, but I find B's chances uncertain. If I list A as the first preference on my ballot, its weight is reduced before it reaches B. If I omit A, B gets a vote with full weight.”

In short, a Hylland free rider is a voter who omits in his individual ranking completely all those candidates who are certain to be elected. Of course, when too many voters use Hylland free riding then it can happen that the candidate with the cast first preference is elected while the candidate with the sincere first preference is eliminated. However, when a voter uses Hylland free riding

then the candidate with the cast first preference is one of this voter's favorite candidates while when this voter uses Woodall free riding then the candidate with the cast first preference is a candidate who this voter does not want to be elected.

Problem 2 can be circumvented by voting only for those candidates who are believed to be in the race until the final count. In so far as a candidate will be in the final count when he has more than  $V/(S+2)$  first preferences, where  $V$  is the number of voters and  $S$  is the number of seats, it is a useful strategy to cast one's first preference only for one of those candidates who are believed to get between  $V/(S+2)$  and  $V/(S+1)$  first preferences.

This voting behaviour could best be observed in Canada because here the city councils were elected for a one year term and in a single city-wide district so that the voters had very precise information about the support of the different candidates. A consequence of this voting behaviour was that usually almost all first preferences were concentrated on  $S+1$  almost equally strong candidates [8, 9, 10]. Johnston [9] writes that one of the main criticisms of STV was that it was “one of the most common features of PR in Canadian municipal elections” that “the final count closely mirrored the results of the first count”. And Pilon [10] writes that the main problem of STV in Canada was that it “did not seem to make much difference in the results. After days of counting, eliminating candidates, and transferring fractions of support from one aspirant to another, there was little difference between the first choice results and the final tally.”

### 4 Summary

Free riding is a very serious problem of STV. The two free riding strategies that have been predicted in the literature are Woodall free riding [1, 2] and Hylland free riding [7]. It is not possible to extract the number of Hylland free riders simply from the ballot data. But with additional assumptions it is possible to extract the number of Woodall free riders.

I used the ballot data of the 1999 and the 2001 City Council elections and School Committee elections in Cambridge, Massachusetts, to estimate the number of voters who use Woodall free riding. I could not find any evidence at all that voters use this strategy. Possible explanations why voters do not use this strategy are:



1. When too many voters cast a first preference for candidate A, not because he is their sincere first preference but because they believe that he will be eliminated early in the count, it could happen that this candidate gets so many votes that he is elected [2, 5, 6].
2. It is not useful to vote for a candidate A who is eliminated with a great probability, because it could happen that there are not acceptable candidates anymore to whom this voter could transfer his vote when candidate A is eliminated.
3. When a voter considers his second favorite candidate to be only slightly worse than his favorite candidate then Hylland free riding [7] is less dangerous than Woodall free riding in so far as a backfire is less severe under Hylland free riding than under Woodall free riding.
4. The political organizations have not yet found a simple way to use Woodall free riding on a larger scale to increase their numbers of seats. Therefore, the voters are usually not pointed to this strategic problem.

- [7] Aanund Hylland, *Proportional Representation without Party Lists*, pp. 126-153, RATIONALITY AND INSTITUTIONS, eds. Raino Malnes and Arild Underdal, Scandinavian University Press, Oslo, 1992
- [8] Joseph P. Harris, *Practical Workings of Proportional Representation in the United States and Canada*, NATIONAL MUNICIPAL REVIEW, vol. 19, pp. 337-383, 1930
- [9] J. Paul Johnston, Miriam Koene, *Learning History's Lessons Anew: The Use of STV in Canadian Municipal Elections*, pp. 205-247, ELECTIONS IN AUSTRALIA, IRELAND, AND MALTA UNDER THE SINGLE TRANSFERABLE VOTE, eds. Shaun Bowler and Bernard Grofman, University of Michigan Press, Ann Arbor, 2000
- [10] Dennis M. Pilon, *The Drive for Proportional Representation in British Columbia, 1917-23*, Simon Fraser University, Vancouver, 1994

## 5 References

- [1] Douglas R. Woodall, *Computer counting in STV elections*, REPRESENTATION, vol. 23, pp. 4-6, 1983 {reprinted in VOTING MATTERS, issue 1, 1994}
- [2] T. Nicolaus Tideman, Daniel Richardson, *Better voting methods through technology: The refinement-manageability trade-off in the single transferable vote*, PUBLIC CHOICE, vol. 103, pp. 13-34, 2000
- [3] Brian L. Meek, *Une nouvelle approche du scrutin transférable*, MATHÉMATIQUES ET SCIENCES HUMAINES, vol. 25, pp. 13-23, 1969 {reprinted in VOTING MATTERS, issue 1, 1994}
- [4] C. H. E. Warren, *Counting in STV Elections*, VOTING MATTERS, issue 1, 1994
- [5] R. J. C. Fennell, *Is a feedback method of calculating the quota really necessary?*, VOTING MATTERS, issue 2, 1994
- [6] I. D. Hill, *Are better STV rules worthwhile? – A reply to R.J.C. Fennell*, VOTING MATTERS, issue 3, 1994

| Election              | Candidate B          | V      | V <sub>1</sub> (B) | V <sub>1</sub> (B)/V | V <sub>2</sub> (B) | V <sub>2</sub> (B)/V |
|-----------------------|----------------------|--------|--------------------|----------------------|--------------------|----------------------|
| 1999 City Council     | Anthony D. Galluccio | 18,613 | 2,705              | 14.5%                | 2,515              | 13.5%                |
| 1999 School Committee | Alice L. Turkel      | 17,796 | 2,617              | 14.7%                | 2,360              | 13.3%                |
| 2001 City Council     | Henrietta Davis      | 17,125 | 1,713              | 10.0%                | 1,645              | 9.6%                 |
| 2001 City Council     | Brian Murphy         | 17,125 | 1,716              | 10.0%                | 1,627              | 9.5%                 |
| 2001 City Council     | Anthony D. Galluccio | 17,125 | 3,230              | 18.9%                | 2,947              | 17.2%                |
| 2001 School Committee | Joseph G. Grassi     | 16,488 | 2,135              | 12.9%                | 1,728              | 10.5%                |
| 2001 School Committee | Alfred B. Fantini    | 16,488 | 2,854              | 17.3%                | 2,353              | 14.3%                |
| 2001 School Committee | Alice L. Turkel      | 16,488 | 2,862              | 17.4%                | 2,484              | 15.1%                |

Table 2.2: V<sub>2</sub> (B)/V for each candidate B who is elected before candidates have to be eliminated

| Candidate A                 | V <sub>1</sub> (A) | Anthony D. Galluccio |
|-----------------------------|--------------------|----------------------|
| Charles O. Christenson      | 28                 | 2 (7.1%)             |
| Daejanna P. Wormwood-Malone | 28                 | 0 (0.0%)             |
| William C. Jones            | 31                 | 2 (6.5%)             |
| Alan Kingfish Nidle         | 40                 | 0 (0.0%)             |
| Vincent Lawrence Dixon      | 44                 | 3 (6.8%)             |
| Jeffrey Jay Chase           | 102                | 10 (9.8%)            |
| Dorothy M. Giacobbe         | 109                | 22 (20.2%)           |
| James M. Williamson         | 128                | 2 (1.6%)             |
| Robert Winters              | 301                | 27 (9.0%)            |
| Helder Peixoto              | 308                | 46 (14.9%)           |
| David Hoicka                | 325                | 7 (2.2%)             |
| Erik C. Snowberg            | 425                | 12 (2.8%)            |
| David Trumbull              | 533                | 129 (24.2%)          |
| Bob Goodwin                 | 805                | 296 (36.8%)          |
| David P. Maher              | 1,030              | 309 (30.0%)          |
| Katherine Triantafillou     | 1,167              | 42 (3.6%)            |
| Michael A. Sullivan         | 1,321              | 278 (21.0%)          |
| Kenneth E. Reeves           | 1,420              | 149 (10.5%)          |
| Henrietta Davis             | 1,458              | 70 (4.8%)            |
| Jim Braude                  | 1,480              | 50 (3.4%)            |
| Timothy J. Toomey, Jr.      | 1,497              | 233 (15.6%)          |
| Marjorie C. Decker          | 1,642              | 43 (2.6%)            |
| Kathleen Leahy Born         | 1,658              | 100 (6.0%)           |

Table 2.3: Potential Woodall free riders in the 1999 City Council elections in Cambridge, Massachusetts

Schulze: Free riding

| Candidate A           | V <sub>1</sub> (A) | Alice L. Turkel |
|-----------------------|--------------------|-----------------|
| Shawn M. Burke        | 212                | 6 (2.8%)        |
| Jamisean F. Patterson | 278                | 9 (3.2%)        |
| Alvin E. Thompson     | 373                | 35 (9.4%)       |
| Melody L. Brazo       | 471                | 82 (17.4%)      |
| Donald Harding        | 698                | 24 (3.4%)       |
| Elizabeth Tad Kenney  | 738                | 134 (18.2%)     |
| Michael Harshbarger   | 1,550              | 109 (7.0%)      |
| Nancy Walser          | 1,894              | 520 (27.5%)     |
| Susana M. Segat       | 1,985              | 480 (24.2%)     |
| Joseph G. Grassi      | 2,269              | 97 (4.3%)       |
| Alfred B. Fantini     | 2,277              | 55 (2.4%)       |
| Denise Simmons        | 2,408              | 506 (21.0%)     |

Table 2.4: Potential Woodall free riders in the 1999 School Committee elections in Cambridge, Massachusetts

| Candidate A            | V <sub>1</sub> (A) | Henrietta Davis | Brian Murphy | Anthony D. Galluccio | Sum (Galluccio, Murphy, Davis) |
|------------------------|--------------------|-----------------|--------------|----------------------|--------------------------------|
| James M. Williamson    | 58                 | 2 (3.4%)        | 2 (3.4%)     | 3 (5.2%)             | 7 (12.1%)                      |
| James E. Condit, III   | 63                 | 6 (9.5%)        | 0 (0.0%)     | 5 (7.9%)             | 11 (17.5%)                     |
| Helder Peixoto         | 69                 | 5 (7.2%)        | 3 (4.3%)     | 7 (10.1%)            | 15 (21.7%)                     |
| Vincent Lawrence Dixon | 92                 | 2 (2.2%)        | 3 (3.3%)     | 7 (7.6%)             | 12 (13.0%)                     |
| Robert L. Hall         | 153                | 3 (2.0%)        | 13 (8.5%)    | 18 (11.8%)           | 34 (22.2%)                     |
| Jacob Horowitz         | 155                | 14 (9.0%)       | 12 (7.7%)    | 6 (3.9%)             | 32 (20.6%)                     |
| Steven E. Jens         | 278                | 8 (2.9%)        | 5 (1.8%)     | 35 (12.6%)           | 48 (17.3%)                     |
| Steve Iskovitz         | 345                | 29 (8.4%)       | 30 (8.7%)    | 9 (2.6%)             | 68 (19.7%)                     |
| Ethridge A. King       | 378                | 43 (11.4%)      | 46 (12.2%)   | 25 (6.6%)            | 114 (30.2%)                    |
| David P. Maher         | 1,017              | 32 (3.1%)       | 41 (4.0%)    | 304 (29.9%)          | 377 (37.1%)                    |
| John Pitkin            | 1,091              | 222 (20.3%)     | 202 (18.5%)  | 48 (4.4%)            | 472 (43.3%)                    |
| Kenneth E. Reeves      | 1,141              | 72 (6.3%)       | 34 (3.0%)    | 125 (11.0%)          | 231 (20.2%)                    |
| Michael A. Sullivan    | 1,315              | 45 (3.4%)       | 28 (2.1%)    | 316 (24.0%)          | 389 (29.6%)                    |
| Denise Simmons         | 1,339              | 186 (13.9%)     | 137 (10.2%)  | 74 (5.5%)            | 397 (29.6%)                    |
| Timothy J. Toomey, Jr. | 1,402              | 44 (3.1%)       | 11 (0.8%)    | 272 (19.4%)          | 327 (23.3%)                    |
| Marjorie C. Decker     | 1,540              | 298 (19.4%)     | 215 (14.0%)  | 163 (10.6%)          | 676 (43.9%)                    |
| Henrietta Davis        | 1,713              | —               | 254 (14.8%)  | 114 (6.7%)           |                                |
| Brian Murphy           | 1,716              | 343 (20.0%)     | —            | 105 (6.1%)           |                                |
| Anthony D. Galluccio   | 3,230              | 137 (4.2%)      | 90 (2.8%)    | —                    |                                |

Table 2.5: Potential Woodall free riders in the 2001 City Council elections in Cambridge, Massachusetts

| Candidate A          | $V_1(A)$ | Joseph G. Grassi | Alfred B. Fantini | Alice L. Turkel | Sum (Turkel, Fantini, Grassi) |
|----------------------|----------|------------------|-------------------|-----------------|-------------------------------|
| Vincent J. Delaney   | 240      | 23 (9.6%)        | 29 (12.1%)        | 5 (2.1%)        | 57 (23.8%)                    |
| Fred Baker           | 324      | 28 (8.6%)        | 62 (19.1%)        | 9 (2.8%)        | 99 (30.6%)                    |
| Marla L. Erlien      | 1,193    | 21 (1.8%)        | 25 (2.1%)         | 272 (22.8%)     | 318 (26.7%)                   |
| Susana M. Segat      | 1,590    | 61 (3.8%)        | 107 (6.7%)        | 619 (38.9%)     | 787 (49.5%)                   |
| Nancy Walser         | 1,677    | 42 (2.5%)        | 68 (4.1%)         | 596 (35.5%)     | 706 (42.1%)                   |
| Richard Harding, Jr. | 1,689    | 172 (10.2%)      | 156 (9.2%)        | 176 (10.4%)     | 504 (29.8%)                   |
| Alan C. Price        | 1,873    | 41 (2.2%)        | 71 (3.8%)         | 319 (17.0%)     | 431 (23.0%)                   |
| Joseph G. Grassi     | 2,135    | —                | 698 (32.7%)       | 94 (4.4%)       |                               |
| Alfred B. Fantini    | 2,854    | 942 (33.0%)      | —                 | 158 (5.5%)      |                               |
| Alice L. Turkel      | 2,862    | 97 (3.4%)        | 133 (4.6%)        | —               |                               |

Table 2.6: Potential Woodall free riders in the 2001 School Committee elections in Cambridge, Massachusetts

## An odd feature in a real STV election

I. D. Hill

No email available.

Although artificial data can be extremely useful in clearly demonstrating difficulties in election rules, there is also much to be said in favour of looking at real data, particularly where anything odd appears to have happened.

A few years ago, there were 23 candidates in an election for 15 seats, and there were 539 votes. The candidates' names have here been coded as A, B, C, etc.

One voter gave preferences, in order, as: M D L R I J C T B E H A O U F etc. Using Newland and Britton (second edition) rules [1], the last candidate elected was F and the runner-up was V. Amazingly, if that one voter had put V instead of F as 15th preference, V would have been elected and F runner-up. In other words, the election result depended upon that one voter's 15th preference.

There is, of course, nothing wrong with a 15th preference being taken into account. If all previous 14 preferences have been excluded it is right that the 15th preference comes through with a value of 1.0 as if it had been a 1st preference. In this case, though, it came through with a value of 1.0 even though 10 of the earlier preferences were elected. Of those 10, 8 had been elected before that vote reached them and, in accordance with the rules, were "leap-frogged". The other 2, J and T are more remarkable; in each case the vote in question was among those that triggered their election and, being part of the last parcel received, was due to be transferred with a transfer value. For both of them, however, there were enough non-transferable votes in the parcel that the transfer value came out as 1.0.

When the final transfer was made, V had 30.31 votes, and F had 30.51, so the additional 1.0 was enough to sway the result. The vote had not had to make any contribution to electing the 10 elected candidates named earlier by the voter.

If Meek rules [2] had been used, that 15th preference would still have been reached, but F would have been ahead of V by almost 4 votes and the value attached to the particular vote, because it would have had to contribute a fair share to electing the earlier 10 candidates, would have been only 0.000000905 and would thus have made no difference.

It is pleasing that, as it happened, the correct result was reached by the actual count, but it could so easily have been the wrong one.

It has sometimes been suggested that messing about with such small fractions of votes, which make no difference to the result, is not worth while. There are two answers to that suggestion. The first is that, if the logic of the Meek method is accepted, then either we can follow that logic through, even if it does result in such "messing about with small fractions", which is easy, or we can put in special rules to stop it doing so, which is much more difficult. We should need to consider not only what special rules to adopt in such cases, but also how to determine when to use them. Obviously it makes sense to do the easy, and correct, thing.

The second answer is that there are cases where such a very small difference can change the answer, so it would be wrong to ignore a 15th preference. If the contest between V and F had reached an exact tie from all the other relevant votes, then the result should, of course, have been settled by what that 15th preference was.

### 1 References

- [1] R A Newland and F S Britton. *How to conduct an election by the Single Transferable Vote*. 2nd edition. Electoral Reform Society. 1976.
- [2] I. D. Hill, B. A. Wichmann and D. R. Woodall. Algorithm 123 — Single Transferable Vote by Meek's method. *Computer Journal* Vol 30, pp271-281. 1987.

## Full disclosure of data

I. D. Hill

No email available.

The objection to full disclosure made by Otten [1] is valid, but seems to me to be of only minor importance. Considering the huge advantages of disclosure, in giving transparency and allowing anyone who wishes to check the result of the counting, it would be a great pity if Otten's point were allowed to prevail over it.

Disclosure does not in itself give complete transparency of the electoral process, because it takes as given the list of votes and their preferences, but in dealing fully with the second part of the process, the counting of the votes, it is nevertheless of great merit.

Otten's "preferred solution" — to suppress later preferences until there are at least three votes of every published pattern — would undoubtedly be better than not publishing the data at all, but it is a very poor thing compared with full disclosure and would, in many instances, lead to the suppression of the very information that would be of importance.

Taking as an example the election reported on in the preceding paper (Hill [2]), the original votes, which had 531 different preference patterns from the 539 votes, would have been reduced to only 96 different patterns, and these would not have shown the vital information that led to the allocation of the final seat. Indeed the 16 votes that put candidate M first would have been shown as just 13 M ... and 3 M R ... The voter whose 15th preference was vital would not have had even a second preference shown.

In an election where political parties were important, it would seem likely that the loss of information would be less severe. Even in the given case, the fact that there were 7 votes starting Q P O S E F H A D J M C B R, and another 3 also starting Q P O S E F, still comes through, indicating obvious collusion between voters (which is not illegal, or even immoral, if that is what they wish to do).

Implementing the Otten procedure is not straightforward, as it is not sufficiently defined. For example, there were 2 votes starting W U A I D, 1 starting W U A I O, 1 starting W U A E. Should these be shown as 4 of W U A ... , or as 3 W U A I ... leaving the other 1 to go in with W ... ? It is not self-evident.

There are many things in life that could be so much simpler if only we could trust everybody, and did not need to bother about fraudsters, but we always need to consider whether a particular fraud is likely, and whether procedures to stop it are doing more harm than good. My personal view is that Otten's suggestion would be doing so.

### 1 References

- [1] Otten J. Fuller disclosure than intended. *Voting matters*, Issue 17, p8. 2003.
- [2] Hill I.D. An odd feature in a real STV election. *Voting matters*, Issue 18, p9. 2004.

# A note on the use of preferences

B A Wichmann  
brian.wichmann@bcs.org.uk

breviated to give only the gender and position in the tables). The election stages were as follows:

## 1 Introduction

With STV, the voter is encouraged to specify as many preferences as may be needed to reflect his/her wishes. The number of preferences actually used within the count is quite a different matter which is the main subject of this note.

For the three Irish constituencies for which a trial was undertaken in 2002 of electronic voting, we have full disclosure of the preferences specified by the voters. This provides an opportunity to analyse the use of preferences in a large public election in some depth.

Joe Otten has stated reservations about the full disclosure of preferential voting data on the grounds that it could allow bribery to take place even though the voting is secret [1]. The issue has also been raised by the Irish Commission on Electronic Voting [3].

Here, we consider how the voter's preferences are used and propose alternative solutions to the problem of disclosure.

## 2 The use of the voter's preferences

It is clear that any preference listed after a continuing candidate cannot be used at that stage of the count. To inspect such a preference would contravene one of the principles of STV. A particular example of this is that those voters who gave their first preference for a candidate who is still a continuing candidate at the end of the count, will not have anything other than their first preference used.

As an example of how preferences are used, consider the 2002 Dáil election for the Meath constituency for which we have full election data. There were 14 candidates for 5 seats (the candidate names have been ab-

|         |                      |
|---------|----------------------|
| Stage 1 | Elect M4             |
| Stage 2 | Exclude F3 and M11   |
| Stage 3 | Exclude M9           |
| Stage 4 | Exclude M8           |
| Stage 5 | Exclude M10          |
| Stage 6 | Exclude M14          |
| Stage 7 | Exclude M6           |
| Stage 8 | Exclude M7, Elect M2 |
| Stage 9 | Elect M1, M5 and F13 |

Hence the continuing candidate is M12.

Now consider an actual voter whose preferences were as follows:

M9 M8 M7 M10 M12 M11 M14 F3 F13 M1 M4 M2 M6 M5

Consulting the actions of the stages above, it is clear that the preferences for M10 and all those after M12 were never used. In other words, the voter could just as well have voted: M9, M8, M7, M12. The other preferences were *invisible*.

To understand the use of the preferences in more detail, we look at the result sheet in Table 5.1. At the second stage, the surplus of M4 is transferred. To do this, all of the 11,534 votes for M4 are inspected and the number whose second preference is given is found, together with the proportion for each of the remaining 13 candidates. Since 853 votes must be transferred to reduce M4 to the quota, an integer is computed for each candidate giving the correct proportion and total. As an example of a transfer, only one vote is transferred to M11 and that vote is selected at random from those giving M11 as the second preference. This implies that 10,681 votes are inspected for their subsequent preference and a further 853 votes are used in the subsequent stages.

Hence we have two uses of preferences with the Irish rules: those used directly to attempt to elect a candidate and those used indirectly to determine which papers to select at random to transfer. For the Meath election, the number of preferences used directly are those for the first preference (the total vote of 64,081) plus the number of those in the table with a + sign but ignoring those in the non-transferable row. The indirect use, which only arises from a transfer of surplus is therefore only from M4, i.e. the 10,681 mentioned above.

In contrast to this, the Meek method uses all the visible preferences. Our sample ballot paper above had four visible preferences M9, M8, M7 and finally M12. In fact, the Irish rules would use all these preferences.

We can now compute the use of the preferences for the three Irish constituencies, expressed as an average per vote:

| Constituency        | Irish-direct | Indirect | Meek | All  |
|---------------------|--------------|----------|------|------|
| Meath               | 1.19         | 0.17     | 1.98 | 4.65 |
| Dublin North        | 1.33         | 0.01     | 2.12 | 4.98 |
| Dublin West         | 1.26         | 0.25     | 2.11 | 4.43 |
| <i>Average of 3</i> | 1.26         | 0.14     | 2.07 | 4.68 |

Hence, as a percentage of all the preferences given, the direct use with the Irish rules is 27%, indirect usage is 3%, while Meek uses 44%.

### 3 Full disclosure?

We can now see that relatively few preferences are actually used in a count. If the voter specifies a large number of preferences, then it is unusual for them all to be used. For an example of a large number of preferences which were used, see [2].

We now have a means of providing an approximation to full disclosure which would nevertheless allow the voter to check the actual count: remove some (or all) of the invisible preferences. For long preference lists, like the one shown above, it would usually be the case that many preferences would be invisible. Hence this strategy of providing full disclosure only of the visible preferences would effectively prohibit the potential problem identified by Joe Otten.

Note that the identification of the invisible preferences depends upon the order of the exclusions and elections which in turn depends upon the particular counting rules being used. Hence, if data were provided with only the visible preferences, then running

that data using a different counting rule would not necessarily give the same result as using the actual data.

## 4 Conclusions

Since many preferences are not used in a count, it is possible to disclose all the used preferences and remove all or part of the unused preferences to avoid any potential breach of confidentiality. The referee made two additional points: it is possible to *add* invisible preferences as well as removing them; and that *any* change to the data implies that a check is not an exact check.

## 5 References

- [1] J Otten. Fuller Disclosure than Intended. *Voting matters*. Issue 17. p 8. 2003.
- [2] I D Hill. What would a different method have done? *Voting matters*. Issue 16. p 5. 2003.
- [3] Interim Report of the Commission on Electronic Voting on the Secrecy, Accuracy and Testing of the chosen Electronic Voting System. [http://www.cev.ie/htm/report/download\\_report.htm](http://www.cev.ie/htm/report/download_report.htm)



|            |       | Surplus<br>M4 | Exclude<br>F3+M11 | Exclude<br>M9 | Exclude<br>M8 | Exclude<br>M10 | Exclude<br>M14 | Exclude<br>M6  | Exclude<br>M7  |
|------------|-------|---------------|-------------------|---------------|---------------|----------------|----------------|----------------|----------------|
| <i>M1</i>  | 8493  | +258<br>8751  | +36<br>8787       | +46<br>8833   | +46<br>8879   | +108<br>8987   | +123<br>9110   | +467<br>9577   | +299<br>9876   |
| <i>M2</i>  | 7617  | +76<br>7693   | +32<br>7725       | +155<br>7880  | +241<br>8121  | +333<br>8454   | +694<br>9148   | +1733<br>10881 | 10881          |
| F3         | 263   | +2<br>265     | -265<br>—         | —             | —             | —              | —              | —              | —              |
| <i>M4</i>  | 11534 | -853<br>10681 | 10681             | 10681         | 10681         | 10681          | 10681          | 10681          | 10681          |
| <i>M5</i>  | 5958  | +61<br>6019   | +52<br>6071       | +68<br>6139   | +126<br>6265  | +374<br>6639   | +737<br>7376   | +1349<br>8725  | +1429<br>10154 |
| M6         | 3877  | +15<br>3892   | +11<br>3903       | +34<br>3937   | +41<br>3978   | +74<br>4052    | +221<br>4273   | -4273<br>—     | —              |
| M7         | 3722  | +29<br>3751   | +56<br>3807       | +113<br>3920  | +185<br>4105  | +359<br>4464   | +675<br>5139   | +119<br>5258   | -5258<br>—     |
| M8         | 1373  | +7<br>1380    | +23<br>1403       | +163<br>1566  | -1566<br>—    | —              | —              | —              | —              |
| M9         | 1199  | +3<br>1202    | +42<br>1244       | -1244<br>—    | —             | —              | —              | —              | —              |
| M10        | 2337  | +16<br>2353   | +53<br>2406       | +224<br>2630  | +200<br>2830  | -2830<br>—     | —              | —              | —              |
| M11        | 180   | +1<br>181     | -181<br>—         | —             | —             | —              | —              | —              | —              |
| M12        | 6042  | +51<br>6093   | +51<br>6144       | +123<br>6267  | +118<br>6385  | +325<br>6710   | +412<br>7122   | +226<br>7348   | +732<br>8080   |
| <i>F13</i> | 8759  | +313<br>9072  | +32<br>9104       | +180<br>9284  | +361<br>9645  | +362<br>10007  | +254<br>10261  | +113<br>10374  | +1261<br>11635 |
| M14        | 2727  | +21<br>2748   | +21<br>2769       | +75<br>2844   | +120<br>2964  | +631<br>3595   | -3595<br>—     | —              | —              |
| Non-T      | —     | —             | +37<br>37         | +63<br>100    | +128<br>228   | +264<br>492    | +479<br>971    | +266<br>1237   | +1537<br>2774  |
| Totals     | 64081 | 64081         | 64081             | 64081         | 64081         | 64081          | 64081          | 64081          | 64081          |

Table 5.1: Meath, 2002: Quota: 10681. Those elected have their names in italics.

# Tie-Breaking with the Single Transferable Vote

Jeffrey C. O’Neill  
jco8@cornell.edu

## 1 Introduction

In tallying the single-transferable vote (STV), ties can occur for several different reasons. With the ERS97 rules [1] for implementing STV, ties can occur when choosing a surplus to transfer (5.2.3), when choosing a candidate to eliminate (5.2.5), and when choosing winners (5.6.2). To illustrate, Table 6.1 shows an example tally with the ERS97 rules. At stage 4, we need to eliminate the candidate with the fewest number of votes, but both C and D are tied for last place.

When ties occur, they need to be broken. One could simply break the tie by lot. However, since there is other information available in an STV count, one can use this information to break the tie. The following are four possible tie-breaking rules.

1. Forwards Tie-Breaking: Choose the candidate who has the most [least] votes at the first stage or at the earliest point in the count where they had unequal votes.
2. Backwards Tie-Breaking: Choose the candidate who has the most [least] votes at the previous stage or at the latest point in the count where they had unequal votes.
3. Borda Tie-Breaking: Choose the candidate with the highest [lowest] Borda score. See [2].
4. Coombs Tie-Breaking: Choose the candidate with the fewest [most] last place votes.

It is possible that after applying one of these tie-breaking rules that the candidates would still be tied. Because of this, it is useful to distinguish between “weak ties” and “strong ties.” A weak tie occurs when candidates have the same number of votes at a given stage. A strong tie occurs when candidates are still tied

after applying a tie-breaking rule, such as one of the four listed above. A strong tie would be broken by lot.<sup>1</sup>

The ERS97 rules use forwards tie-breaking. The purpose of this paper is twofold. First, to show that backwards tie-breaking is a better solution and to suggest that the ERS97 rules be changed to use backwards tie-breaking instead. Second, to show that substage totals should not be used when breaking ties.

## 2 Backwards or Forwards Tie-Breaking

In breaking a tie, the ERS97 rules state that one must choose “the candidate who had the greatest vote [or fewest votes] at the first stage or at the earliest point in the count, after the transfer of a batch of papers, where they had unequal votes.” This is forwards tie-breaking and is used when choosing a surplus to transfer (5.2.3), when choosing a candidate to eliminate (5.2.5), and when choosing winners (5.6.2).

The difference between backwards and forwards tie-breaking will be illustrated with the example in Table 6.1. In this example, we have to eliminate one candidate at stage 4 and there is a weak tie between candidates C and D. Thus, tie-breaking needs to be used to determine which candidate is to be eliminated. Under ERS97 rules, we break the tie by using forwards tie-breaking. To do this we first look to the counts at stage 1. We see that D has one more vote than C at stage 1. Thus, candidate C is eliminated.<sup>2</sup>

Another alternative is to use backwards tie-breaking. To do this, we look at the previous stage to break ties, and if necessary to preceding stages. Looking at the

---

<sup>1</sup>Of course one could use another tie-breaking rule if the first tie-breaking rule results in a tie, but this will not be considered here. Borda and Coombs tie-breaking are just presented as available alternatives and will not be discussed further.

<sup>2</sup>If C and D had been tied at stage 1, then we would have looked to subsequent stages. If C and D had been tied at all stages, then we would have had a strong tie which would have been broken by lot.

preceding stage, we see that C is ahead of D at stage 3. Thus, D would be eliminated.

One problem with forwards tie-breaking is that it looks at the stages in an order that is not sequential. In order to determine the candidate to be eliminated at stage 4, we would look at the stages in the following order: 4 1 2 3. Intuitively, this is undesirable. It makes more sense to look at the stages in sequential order. Since one must look first to the current stage, there is only one sequential ordering: 4 3 2 1. This is what backwards tie-breaking would do.

A more important problem, is that forwards tie-breaking does not use the most relevant information to break the tie. The most relevant information to break a tie is the previous stage and not all the way back to the very first stage. By immediately looking to the first stage to break the tie, the ERS97 rules allow the tie-breaking to be influenced by candidates eliminated very early in the process and also by surpluses yet to be transferred. Instead, if we look to the previous stage to break a tie, candidates eliminated early on in the process will have no influence in breaking the tie. In addition, it allows for surpluses to be transferred which gives a more accurate picture of candidate strength.

In Table 6.1, candidate C has more support than candidate D at stage 3. At this point, the surplus of A has already been transferred and candidate F has already been eliminated. Thus, stage 3 is a better measure than is stage 1 as to which candidate should be eliminated at stage 4.

Other implementations of the single transferable vote use backwards tie-breaking instead of forwards tie-breaking: Cambridge, MA STV [3], rules advocated by the Center for Voting and Democracy [4], and rules advocated by the Proportional Representation Society of Australia [5].

### 3 Elimination of Winning Candidates

An incidental problem related to using forwards tie-breaking is that the ERS97 rules can sometimes eliminate a winning candidate. Consider an example where 31 voters elect one candidate with the following ballots:

|    |             |      |
|----|-------------|------|
| 4  | voters vote | ABC  |
| 5  | voters vote | BC   |
| 5  | voters vote | CB   |
| 2  | voters vote | DABC |
| 4  | voters vote | EABC |
| 11 | voters vote | F    |

Table 6.2 shows the results of the tally with ERS97 rules.

At stage 3 of the count, we need to eliminate one or more candidates and candidates B and C are tied with the fewest votes. According to rule 5.2.5(b), both B and C are to be eliminated. However, if instead the tie between B and C was broken by lot, then the other candidate would go on to win the election! In this scenario, suppose candidate C was eliminated by lot at stage three. Then B would be tied with A at stage 4, each with 10 votes. Forwards tie-breaking would be used to break the tie. Candidate A has the fewest votes at stage 1 and would then be eliminated. B would then receive all of A's votes and beat F 20 to 11 in the final stage.

Thus, the ERS97 rules are over-aggressive in eliminating candidates. This is a clear flaw in the ERS97 rules. This flaw arises from the interaction of rule 5.2.5(b) and forwards tie-breaking. This flaw could be fixed in two ways: (1) by changing rule 5.2.5(b), or (2) by using backwards tie-breaking instead of forwards tie-breaking. Since there are already other good reasons for using backwards tie-breaking, the obvious choice is (2).

If backwards tie-breaking were used instead, then both candidates B and C could properly be eliminated at stage 3. If just C were eliminated and B received all of C's votes, then there would again be a tie at stage 4. However, with backwards tie-breaking, B would necessarily have fewer votes than A at the previous stage and would immediately be eliminated.

Backwards tie-breaking would fix this flaw generally, and not just in this specific example. This flaw occurs under specific conditions:<sup>3</sup> (1) a candidate needs to be eliminated and two candidates are tied for last place, (2) the sum of the votes of these two candidates is equal to the candidate with the next fewest number of votes, and (3) after eliminating one of these candidates there would be a subsequent tie with this third candidate. Under these conditions rule 5.2.5(b) requires that the two candidates in last place be eliminated simultaneously. As described above, with forwards tie-breaking a winning candidate could be improperly eliminated. However, with backwards tie-breaking, both of these last-place candidates cannot win and can thus be properly eliminated. The two last-place candidates are guaranteed to lose the second tie because they necessarily

<sup>3</sup>These conditions could be generalized to the case where more than two candidates are tied for last place.

have fewer votes at the previous stage (but they do not necessarily have fewer votes at the first stage).

#### 4 Use of Substages to Break Ties

The word “substage” is not used anywhere in the ERS97 rules, but this terminology is used by people familiar with the rules. Substages can occur when transferring votes from eliminated candidates. Table 6.3 shows an example using ballots from the test T143 where 60 voters are electing two candidates. At stage 3, candidate F is being eliminated. Candidate F has ballots with transfer value 1.00 and ballots with transfer value 0.25 (from the surplus of A). These ballots will be transferred in two substages constituting two different batches. The first substage transfers ballots with value 1.00 and the second transfers ballots with value 0.25.

In stage 4 of this example, we need to eliminate a candidate and candidates C and D are tied for last place. Hence, we need to use forwards tie-breaking. With ERS97 rules, substages must be considered when doing forwards tie-breaking. Candidates C and D are also tied at stage 1 and stage 2, but candidate D is ahead of candidate C at the substage between stages 2 and 3. Thus, candidate C is eliminated.

The problem is that substages are not a good metric for breaking ties. In the example in Table 6.3, either candidate C or D must be eliminated at stage 4. Candidates C and D are tied at stages 4, 1, and 2. Candidate C is ahead at stage 3, but candidate C is eliminated anyway! The reason that C is eliminated is that D has more votes at an intermediary point where only some of candidate F's votes have been transferred. This intermediary point is well-defined but completely arbitrary in terms of fairness. There is no reason to make some of F's votes more important than others. Whether one candidate is ahead of another at this intermediary point is not relevant to which candidate should be eliminated. What is relevant, is what the counts are at each stage of the count, that is after a candidate has been completely eliminated.

#### 5 Conclusions

The ERS97 rules should be changed so that backwards tie-breaking is used instead of forwards tie-breaking. In addition, substage totals should not be considered when breaking ties.

#### 6 Acknowledgments

The author would like to thank David Gamble, Brian Wichmann, and an anonymous reviewer for their valuable suggestions regarding this paper.

#### 7 References

- [1] R A Newland and F S Britton. *How to conduct an election by the Single Transferable Vote*. ERS 3rd Edition. 1997. See <http://www.electoral-reform.org.uk/votingsystems/stvrules.htm>.
- [2] Earl Kitchener, Tie-Breaking in STV. *Voting matters*. Issue 11, April 2000.
- [3] Massachusetts General Laws, Chapter 54A, Section 9(k).
- [4] Choice Voting. The Center for Voting and Democracy. See [http://www.fairvote.org/library/statutes/choice\\_voting.htm](http://www.fairvote.org/library/statutes/choice_voting.htm)
- [5] Rules of the Proportional Representation Society of Australia for conducting elections by the quota-preferential method. 2001. See <http://www.prsa.org.au/rule1977.htm>

|                  |    | Surplus<br>of A | Eliminate<br>F | Eliminate<br>E | Eliminate<br>C |
|------------------|----|-----------------|----------------|----------------|----------------|
| Stage            | 1  | 2               | 3              | 4              | 5              |
| A                | 23 | 20.00           | 20.00          | 20.00          | 20.00          |
| B                | 13 | 13.00           | 13.00          | 15.00          | 15.00          |
| C                | 6  | 6.50            | 10.00          | 12.00          | 2.00           |
| D                | 7  | 7.50            | 9.50           | 12.00          | 18.00          |
| E                | 7  | 7.50            | 7.50           | -              | -              |
| F                | 4  | 5.50            | -              | -              | -              |
| Non-Transferable | 0  | 0.00            | 0.00           | 1.00           | 5.00           |

Table 6.1: Example tally with ERS97 rules where 60 voters are electing two candidates.

|                  |    | Eliminate<br>D | Eliminate<br>E | Eliminate<br>B & C |
|------------------|----|----------------|----------------|--------------------|
| Stage            | 1  | 2              | 3              | 4                  |
| A                | 4  | 6.00           | 10.00          | 10.00              |
| B                | 5  | 5.00           | 5.00           | -                  |
| C                | 5  | 5.00           | 5.00           | -                  |
| D                | 2  | -              | -              | -                  |
| E                | 4  | 4.00           | -              | -                  |
| F                | 11 | 11.00          | 11.00          | 11.00              |
| Non-Transferable | 0  | 0.00           | 0.00           | 10.00              |

Table 6.2: Example where the ERS97 rules eliminate a winning candidate. Thirty-one voters are electing one candidate. Candidate F is the winner.

|                  |    | Surplus<br>of A | Eliminate F | Eliminate<br>E | Eliminate<br>C |       |
|------------------|----|-----------------|-------------|----------------|----------------|-------|
| Stage            | 1  | 2               | substage    | 3              | 4              | 5     |
| A                | 23 | 20.00           | 20.00       | 20.00          | 20.00          | 20.00 |
| B                | 13 | 13.00           | 13.00       | 13.00          | 15.00          | 15.00 |
| C                | 7  | 7.50            | 8.50        | 10.00          | 12.00          | 2.00  |
| D                | 7  | 7.50            | 9.50        | 9.50           | 12.00          | 18.00 |
| E                | 6  | 6.50            | 6.50        | 6.50           | -              | -     |
| F                | 4  | 5.50            | 1.50        | -              | -              | -     |
| Non-Transferable | 0  | 0.00            | 1.00        | 1.00           | 1.00           | 5.00  |

Table 6.3: ERS97 rules with substage tie-breaking. Sixty voters are electing two candidates.